エネルギー環境論

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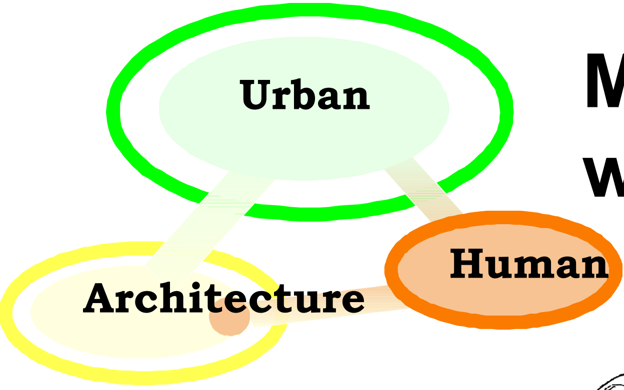
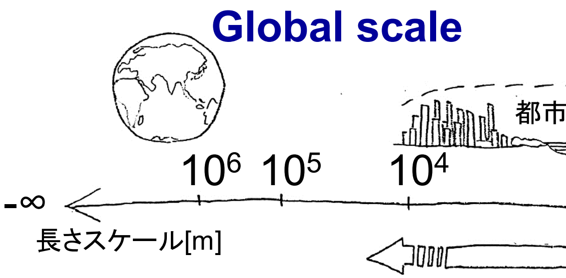
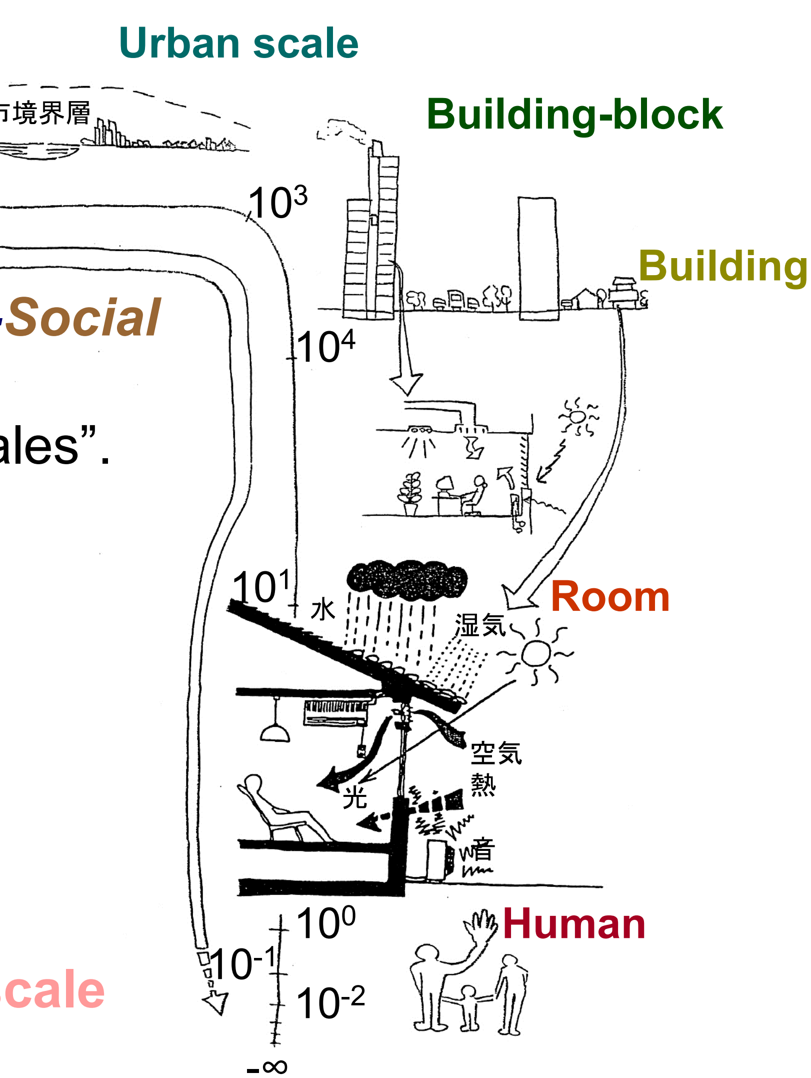
第6回講義

社会ジレンマをモデル化する

ー統計物理学，進化ゲーム理論と社会ジレンマー

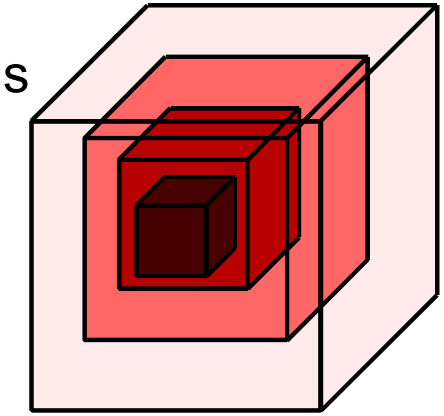


Mutually-interpenetrative view over wide spatial-scales

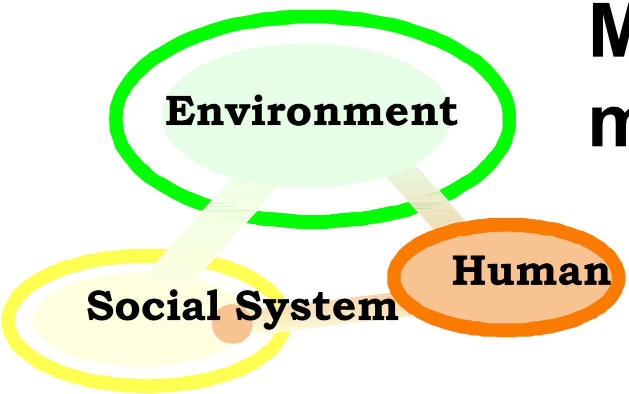
To elaborate the ***Human - Environment -Social System***, it's important a concept of "Simultaneous" or "Bridging to various scales".

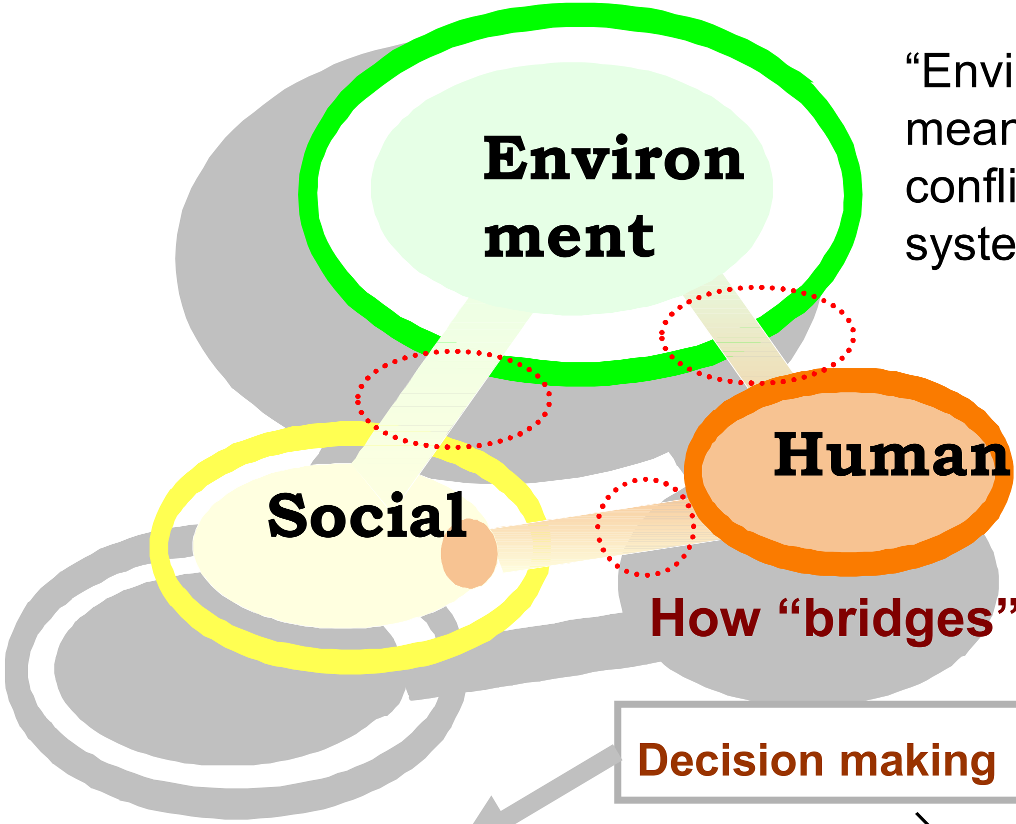
Two physical systems having neighboring special scales are mutually connected through boundary conditions.



Small scale⟵Interaction⟶**Large scale**

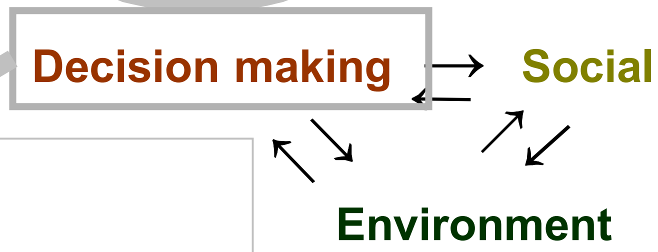
Mutually-interpenetrative view over mutually different systems





"Environmental problems" mean social dilemmas conflicting those three systems.

How "bridges" are defined?



Science for complex system

Evolutionary game theory, Multi-agent simulation, Artificial intelligence (GA, NNw etc)

What is the *Game Theory* ?

Game theory is a study of strategic decision making. More formally, it is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers.“

John von Neumann & Oskar Morgenstern; Theory of games and economic behavior, 1944.

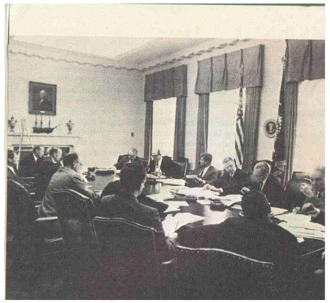
Game theory has been widely recognized as an important tool in many fields; economics, political science, psychology, as well as biology, information science and even statistical physics. Eight game-theorists, including John Nash have won the Nobel Memorial Prize in Economic Sciences, and John Maynard Smith was awarded the Crafoord Prize for his application of game theory to biology.

Zero-sum (Constant-sum) games

(Japanese) Chess, Go. Minimax theorem (von Neumann); For every two-person, zero-sum game with finitely many strategies, there exists a value V and a mixed strategy for each player, such that (a) Given player 2's strategy, the best payoff possible for player 1 is V, and (b) Given player 1's strategy, the best payoff possible for player 2 is −V.

Non zero-sum (Non constant-sum) games

Many applications happening in real world. Social dilemma, Prisoner’s Dilemma, Chicken games etc.



Cuba Crisis -->Chicken game?

2 by 2 game

Agent1

Agent2

	Player 2 chooses *Left*	Player 2 chooses *Right*
Player 1 chooses *Up*	4, 3	–1, –1
Player 1 chooses *Down*	0, 0	3, 4

Normal form or payoff matrix of a 2-player, 2-strategy game

Agent1 \ Agent2	Cooperation (C)	Defection (D)
Cooperation (C)	$R,\ R$	$S,\ T$
Defection (D)	$T,\ S$	$P,\ P$

R; Reward, T; Temptation, S; Sucker, P; Punishment

Application; Analytical approach concerning equilibrium (steady-state) for Nonlinear systems

2-player 2-strategy game (2 by 2 game)

Class	Dilemma?	GID	RAD
Prisoner's Dilemma; PD	Yes	Yes	Yes
Chicken (Snow Drift; Hawk-Dove)	Yes	Yes	No
Stag Hunt; SH	Yes	No	Yes
Trivial	No	No	No

Basic Assumption

- Infinite population.
- One-shot game; well-mixed situation (with neither social viscosity nor assortment among agents).

Prisoner's Dilemma

Agent1 Agent2

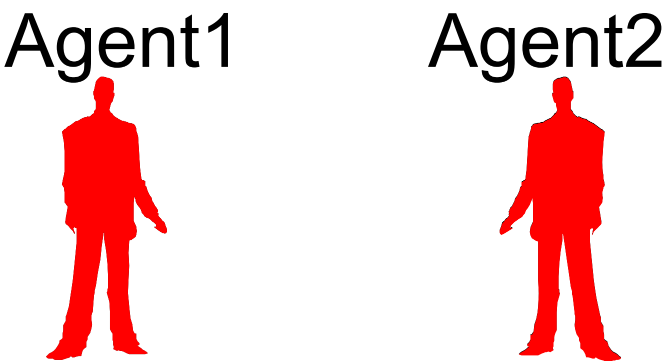
Agent1 \ Agent2	Cooperation (C)	Defection (D)
Cooperation (C)	R, R	S, T
Defection (D)	T, S	P, P

R; <u>R</u>eward, T; <u>T</u>emptation, S; <u>S</u>ucker, P; <u>P</u>unishment

Prisoner's Dilemma

	C	D
C	R, R	S, T
D	T, S	P, P

R; *R*eward, *T*; *T*emptation
S; *S*ucker, *P*; *P*unishment



$2R(=8) > T+S(=6) > 2P(=4)$

Agent2 / Agent1	Cooperation (C)	Defection (D)
Cooperation (C)	5, 5 Equal Pareto Optimum	1, 7
Defection (D)	7, 1	3, 3 Nash Equilibrium

Gamble-Intending Dilemma (GID); $D_g = T - R = 7 - 5 > 0$

Risk-Averting Dilemma (RAD); $D_r = P - S = 3 - 1 > 0$

Prisoner's Dilemma

	C	D
C	R	S
D	T	P

R; Reward, T; Temptation
S; Sucker, P; Punishment

Agent1 Agent2

$2R(=8) > T+S(=6) > 2P(=4)$

Agent1 \ Agent2	Cooperation (C)	Defection (D)
Cooperation (C)	5	1
Defection (D)	7	3

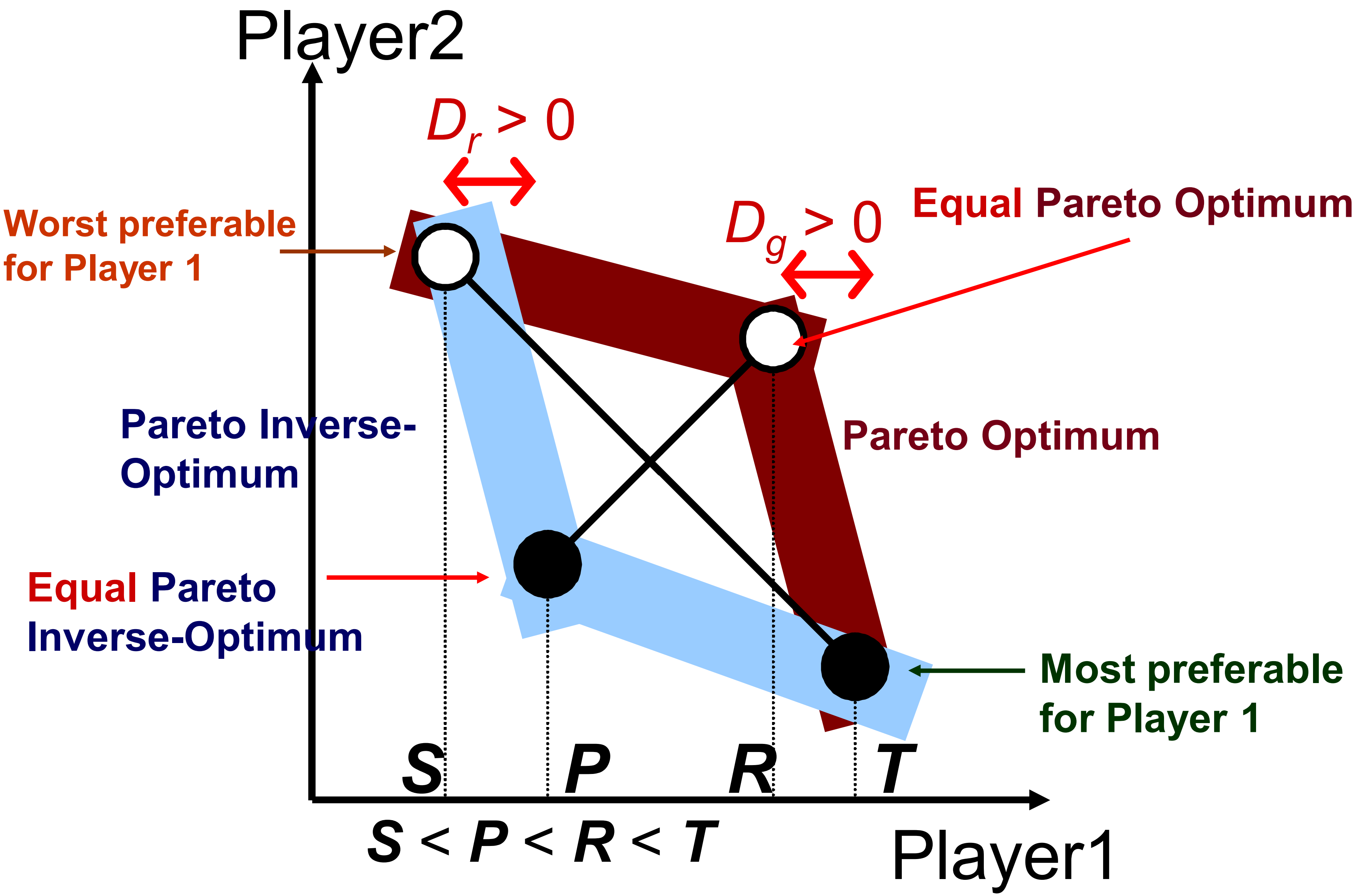
Equal Pareto Optimum

Nash Equilibrium

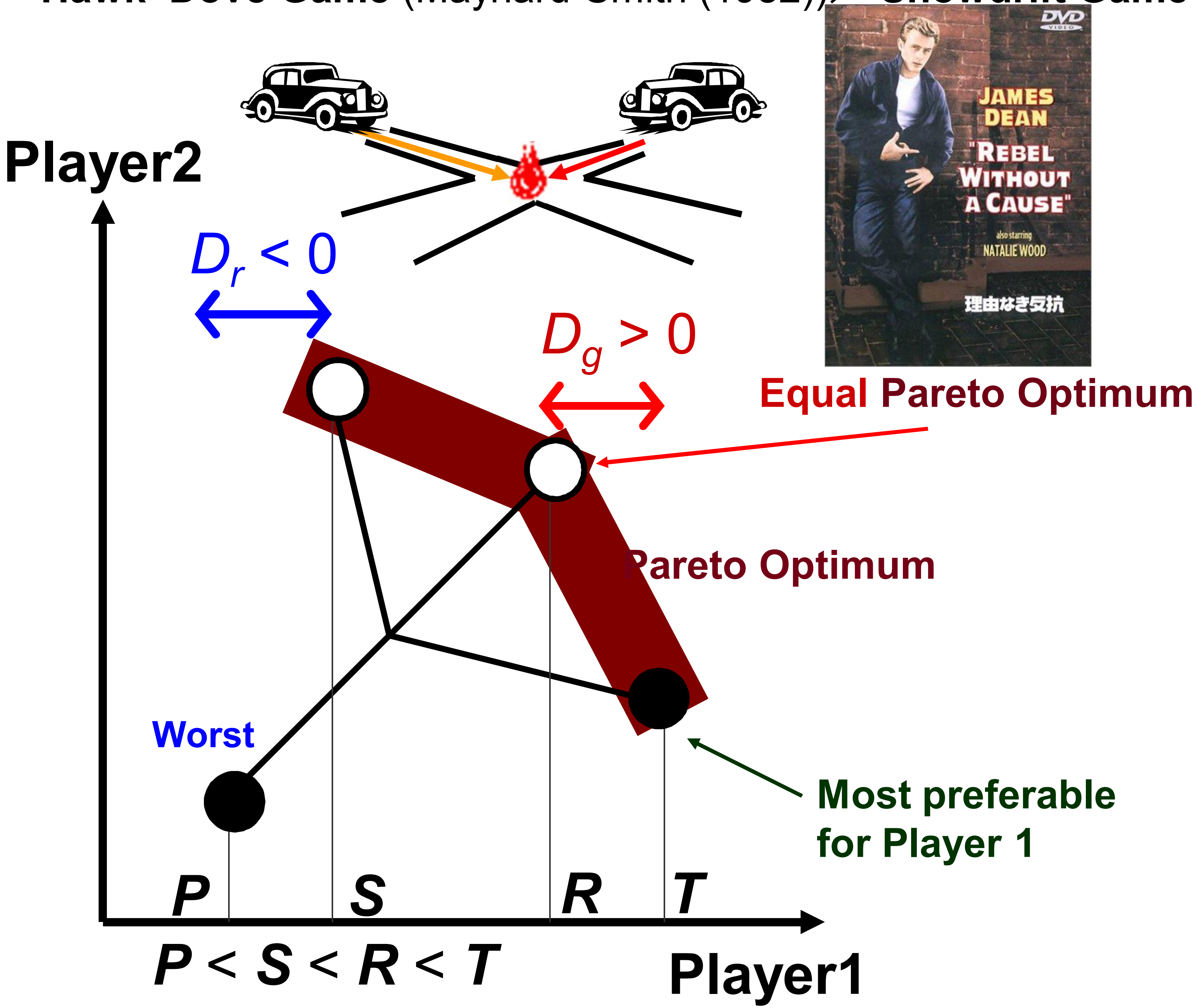
Gamble-Intending Dilemma (GID); $D_g = T - R = 7 - 5 > 0$

Risk-Averting Dilemma (RAD); $D_r = P - S = 3 - 1 > 0$

Prisoner's Dilemma



Chicken／ Hawk–Dove Game (Maynard Smith (1982))／ Snowdrift Game



Chicken

	C	D
C	R	S
D	T	P

R; *R*eward, *T*; *T*emptation
S; *S*ucker, *P*; *P*unishment

Agent1 Agent2

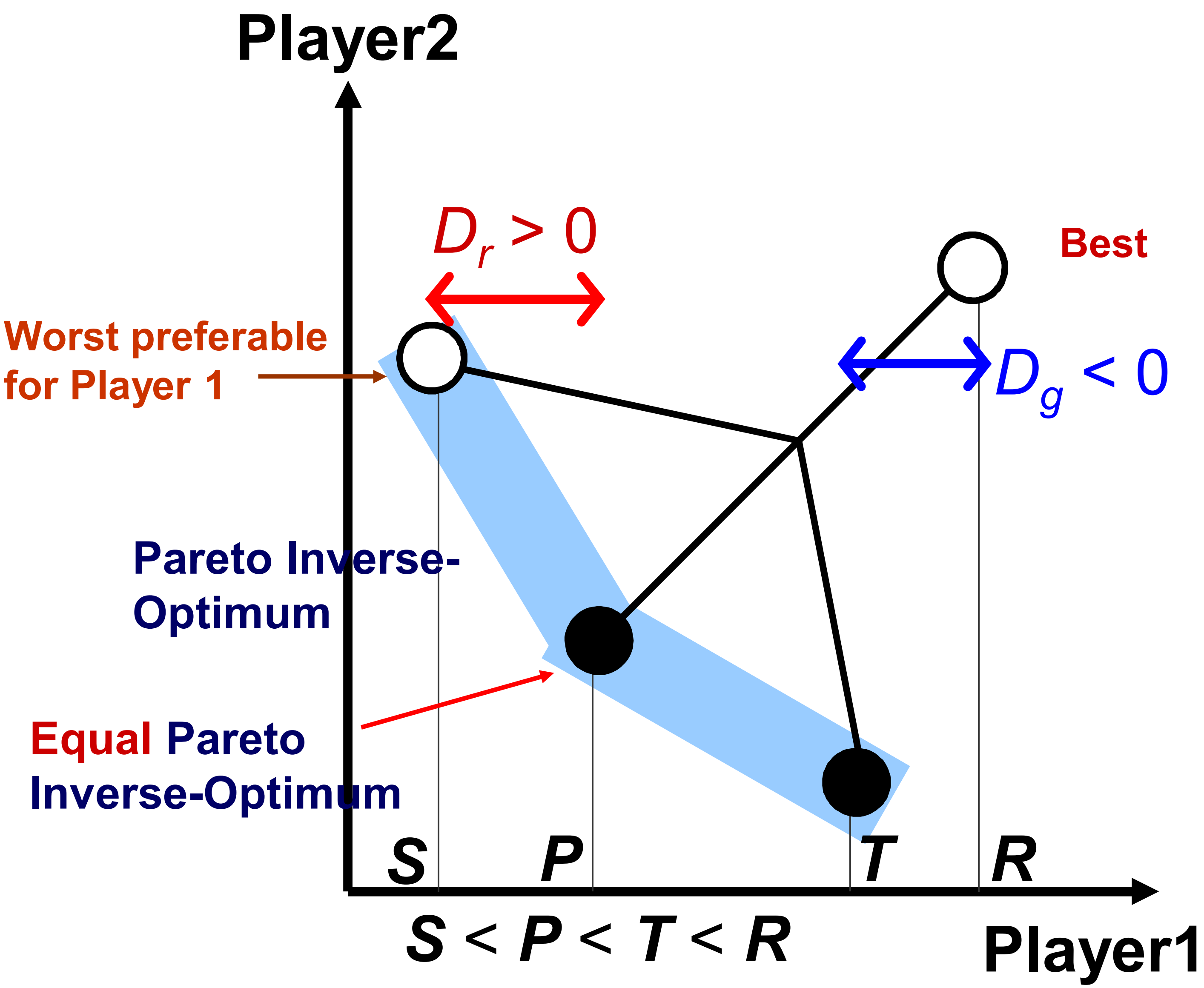

$2R(=8) > T+S(=6) > 2P(=4)$

Agent2 / Agent1	Cooperation (C)	Defection (D)
Cooperation (C)	5 **Equal Pareto Optimum**	3 **Nash Equilibrium**
Defection (D)	7 **Nash Equilibrium**	1 **Worst**

Gamble-Intending Dilemma (GID); $D_g = T - R = 7 - 5 > 0$

Risk-Averting Dilemma (RAD); $D_r = P - S = 3 - 1 < 0$

Stag Hunt ⁄ Inspired by Jean-Jacques Rousseau; “Discours sur l'origine et les fondements de l'inégalité parmi les hommes” (Chapter 2)



Stag Hunt

	C	D
C	R	S
D	T	P

R; *R*eward, *T*; *T*emptation
S; *S*ucker, *P*; *P*unishment

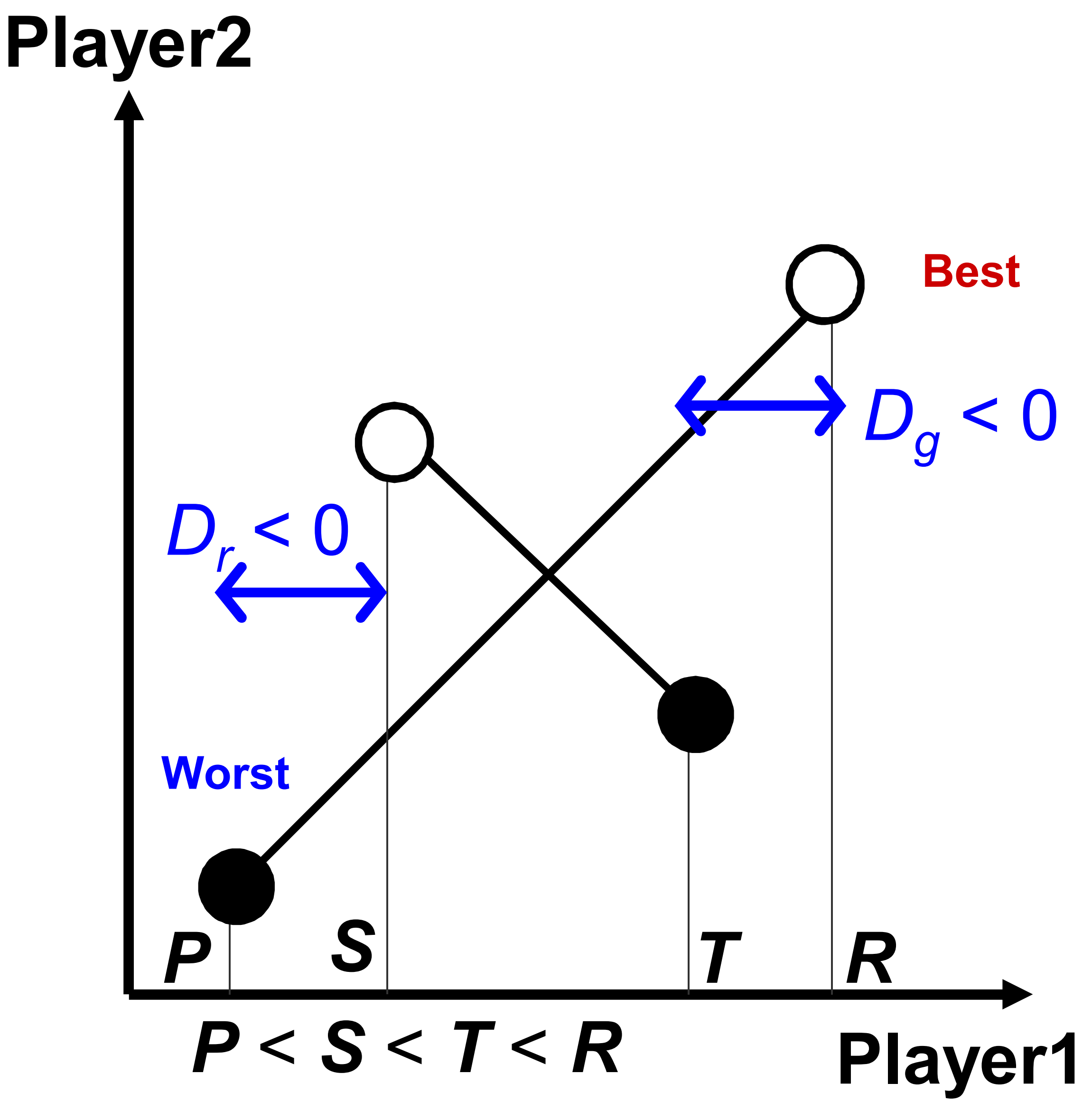
Agent1 Agent2

Agent1 \ Agent2	Cooperation (C)	Defection (D)
Cooperation (C)	Nash Equilibrium 7 Best=Equal Pareto Optimum	1
Defection (D)	5	3 Nash Equilibrium

Gamble-Intending Dilemma (GID); $D_g = T - R = 5 - 7 < 0$

Risk-Averting Dilemma (RAD); $D_r = P - S = 3 - 1 > 0$

Trivial Dilemma Free game



Trivial

	C	D
C	R	S
D	T	P

R; *R*eward, *T*; *T*emptation
S; *S*ucker, *P*; *P*unishment

Agent1 Agent2

Agent2 / Agent1	Cooperation (C)	Defection (D)
Cooperation (C)	Nash Equilibrium 7 Best=Equal Pareto Optimum	3
Defection (D)	5	1

Gamble-Intending Dilemma (GID); $D_g = T - R = 5 - 7 < 0$

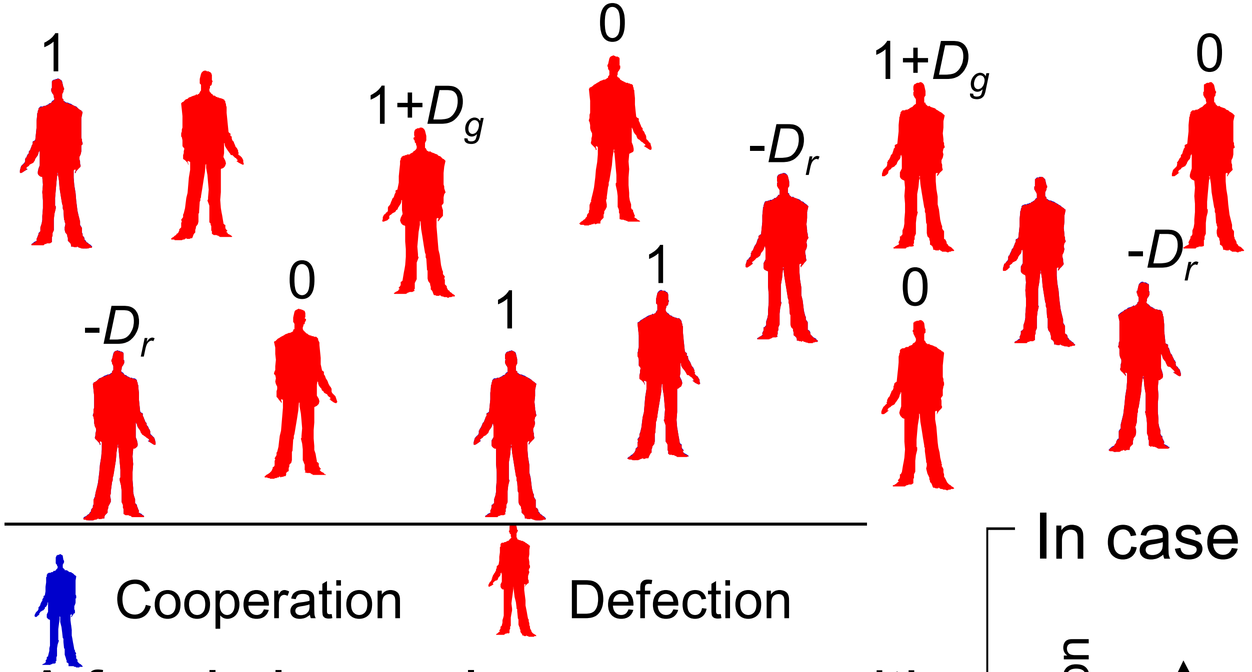
Risk-Averting Dilemma (RAD); $D_r = P - S = 1 - 3 < 0$

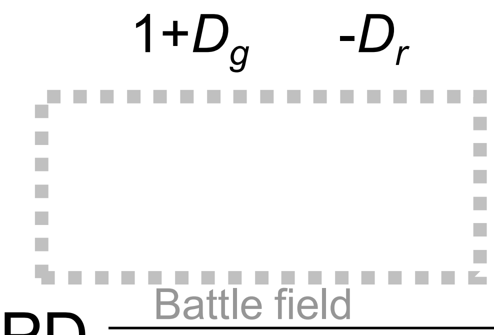
Evolutionary game

2 by 2 game considered time evolution

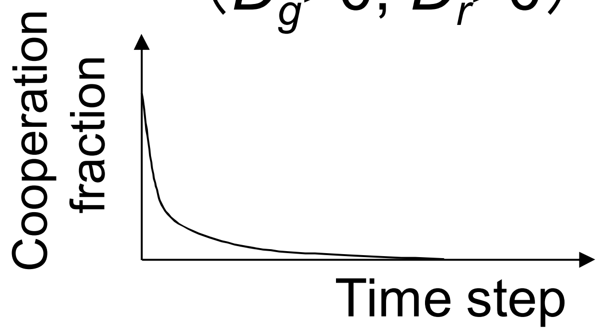
	C	D
C	1	$-D_r$
D	$1+D_g$	0

D_g; GID
D_r; RAD





In case if PD
(D_g>0, D_r>0)



1. A focal player plays a game with a randomly selected opponent.
2. Strategy (whether C or D) adaptation based on obtained payoff is considered.

You never see emerging cooperation, unless some additional mechanism for social viscosity is implemented.

What is *Social Viscosity*?

- Kin selection
- Direct reciprocity
- Indirect Reciprocity
- Network Reciprocity

- Group selection

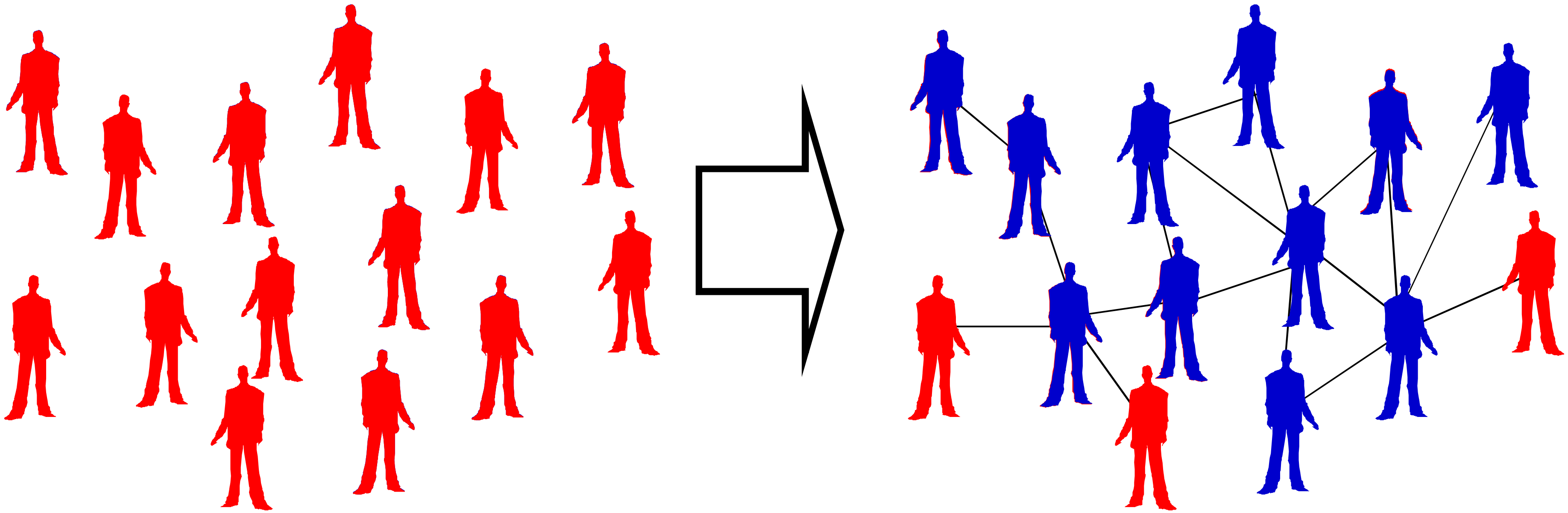
A restricted relation among agents

↓

Lessing *Anonymity*

↓

Emerging cooperation



Well-mixed situation

A Game on a network

Let us back to the Basic Assumption again;

- Infinite population.
- One-shot game; well-mixed situation (with neither social viscosity nor assortment among agents).

Let us describe Cooperation and defection strategies by;

$$ {}^{T}\mathbf{e_1} = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad ; \text{C} $$

$$ {}^{T}\mathbf{e_2} = \begin{pmatrix} 0 & 1 \end{pmatrix} \quad ; \text{D} $$

Also, let us define game structure, i.e. payoff matrix as below;

$$ \begin{bmatrix} R & S \\ T & P \end{bmatrix} \equiv \mathbf{M} $$

Further, let us define strategy frequency among agents at a certain time step as below;

$$ {}^{T}\mathbf{s} = \begin{pmatrix} s_1 & s_2 \end{pmatrix} $$

Fraction of **C** **D**

By simplex constraint; $s_2 = 1 - s_1$.

Let us think a simple example. When a focal player who offers D, how much of payoff expectation she can get in case of paying with another D player as her game opponent?

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \cdot \begin{bmatrix} P & S \\ T & P \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = P$$

By analogy, payoff expectations of both a C and D players respectively paying with average players at this time step are;

$${}^{T}\mathbf{e}_1 \cdot \mathbf{M}\,\mathbf{s}$$

$${}^{T}\mathbf{e}_2 \cdot \mathbf{M}\,\mathbf{s}$$

Let us consider the following system dynamics, called **Replicator Dynamics**, which is thought to be a good model for describing the reproduction process of population dynamics for animal species.

$$\frac{\dot{s}_i}{s_i} = {}^{T}\mathbf{e_i} \cdot \mathbf{M}\,\mathbf{s} - {}^{T}\mathbf{s} \cdot \mathbf{M}\,\mathbf{s}$$

Changing rate of strategy *i*; C when *i*=1 & D when *i*=2

Payoff expectations of a strategy *i* player paying with an average player at this time step

Payoff expectations of an average player paying with an average player at this time step

Implying benefit brought to a player who adopts strategy *i*.

Replicator Dynamics: $\frac{\dot{s}_i}{s_i} = {}^T\mathbf{e_i} \cdot \mathbf{M}\,\mathbf{s} - {}^T\mathbf{s} \cdot \mathbf{M}\,\mathbf{s}$ has three equilibriums.

Two obvious equilibriums are;

$(1,0)$; A state absorbed by C where all players offer C (C Dominate phase) .

$(0,1)$; A state absorbed by D where all players offer D (D Dominate phase) .

The third one is;

(Polymorphic phase).

A question is what dynamics would be if analytic approach is applied to the Replicator Dynamics, which is a (nonlinear) cubic equation for s_1 or s_2.

Let us describe Replicator Dynamics explicitly by substituting *i*=1 and 2.

$$\frac{\dot{s}_i}{s_i} = {}^T\mathbf{e_i} \cdot \mathbf{M}\,\mathbf{s} - {}^T\mathbf{s} \cdot \mathbf{M}\,\mathbf{s}$$

$$\Leftrightarrow \begin{cases} \dot{s}_1 = \left[(R-T)\cdot s_1 - (P-S)\cdot s_2\right]\cdot s_1 \cdot s_2 \\ \dot{s}_2 = -\left[(R-T)\cdot s_1 - (P-S)\cdot s_2\right]\cdot s_1 \cdot s_2 \end{cases}$$

When defining $\dot{s}_1 \equiv f_1(s_1, s_2)$ and $\dot{s}_2 \equiv f_2(s_1, s_2)$ as well as reminding Simplex constraint; $s_2 = 1 - s_1$, we know;

$$f_1 = -f_2$$

Again, Our current target is **to evaluate Eigen values of Jacobi Matrix at respective three equilibrium**; s^*.

$$\mathbf{f}'(\mathbf{x}^*) = \left.\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*} = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_n(\mathbf{x})}{\partial x_n} \end{bmatrix}_{\mathbf{x}=\mathbf{x}^*}$$

$$\begin{cases} \dfrac{\partial f_1}{\partial s_1} = -\dfrac{\partial f_2}{\partial s_1} = 3(-R+S+T-P){s_1}^2 \\ \qquad +2(R-2S-T+2P)s_1 + S - P \\ \\ \dfrac{\partial f_1}{\partial s_2} = -\dfrac{\partial f_2}{\partial s_2} = -3(-R+S+T-P){s_1}^2 \\ \qquad -2(R-2S-T+2P)s_1 - S + P \end{cases}$$

We know two Eaigen values of $\mathbf{J} = \begin{bmatrix} \dfrac{\partial f_1}{\partial s_1} & \dfrac{\partial f_1}{\partial s_2} \\ \dfrac{\partial f_2}{\partial s_1} & \dfrac{\partial f_2}{\partial s_2} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial s_1} & \dfrac{\partial f_1}{\partial s_2} \\ -\dfrac{\partial f_1}{\partial s_1} & -\dfrac{\partial f_1}{\partial s_2} \end{bmatrix}$ are;

0 and $\dfrac{\partial f_1}{\partial s_1} - \dfrac{\partial f_1}{\partial s_2}$ (its eiven vector is $(1,\text{-}1)$) .

Thus, what we should currently do is evaluate sings of $\lambda \equiv \frac{\partial f_1}{\partial s_1} - \frac{\partial f_1}{\partial s_2}$ at respective three equilibrium; s^*.

$$\lambda = \frac{\partial f_1}{\partial s_1} - \frac{\partial f_1}{\partial s_2} = 6(-R + S + T - P)s_1^{\,2} + 4(R - 2S - T + 2P)s_1 + 2(S - P)$$

(1) At $s^* = (1,0)$; $\lambda = -2R + 2T$.

Thus, for $\lambda < 0$, it must be $T - R = D_g < 0$.

(2) At $s^* = (0,1)$; $\lambda = 2S - 2P$.

Thus, for $\lambda < 0$, it must be $P - S = D_r > 0$.

(3) At $s^* = \left(\frac{P-S}{P-T-S+R} \quad \frac{R-T}{P-T-S+R} \right)$; $\lambda = 2\frac{(R-T)(P-S)}{R-S-T+P}$.

Thus, for $\lambda < 0$, it must be;

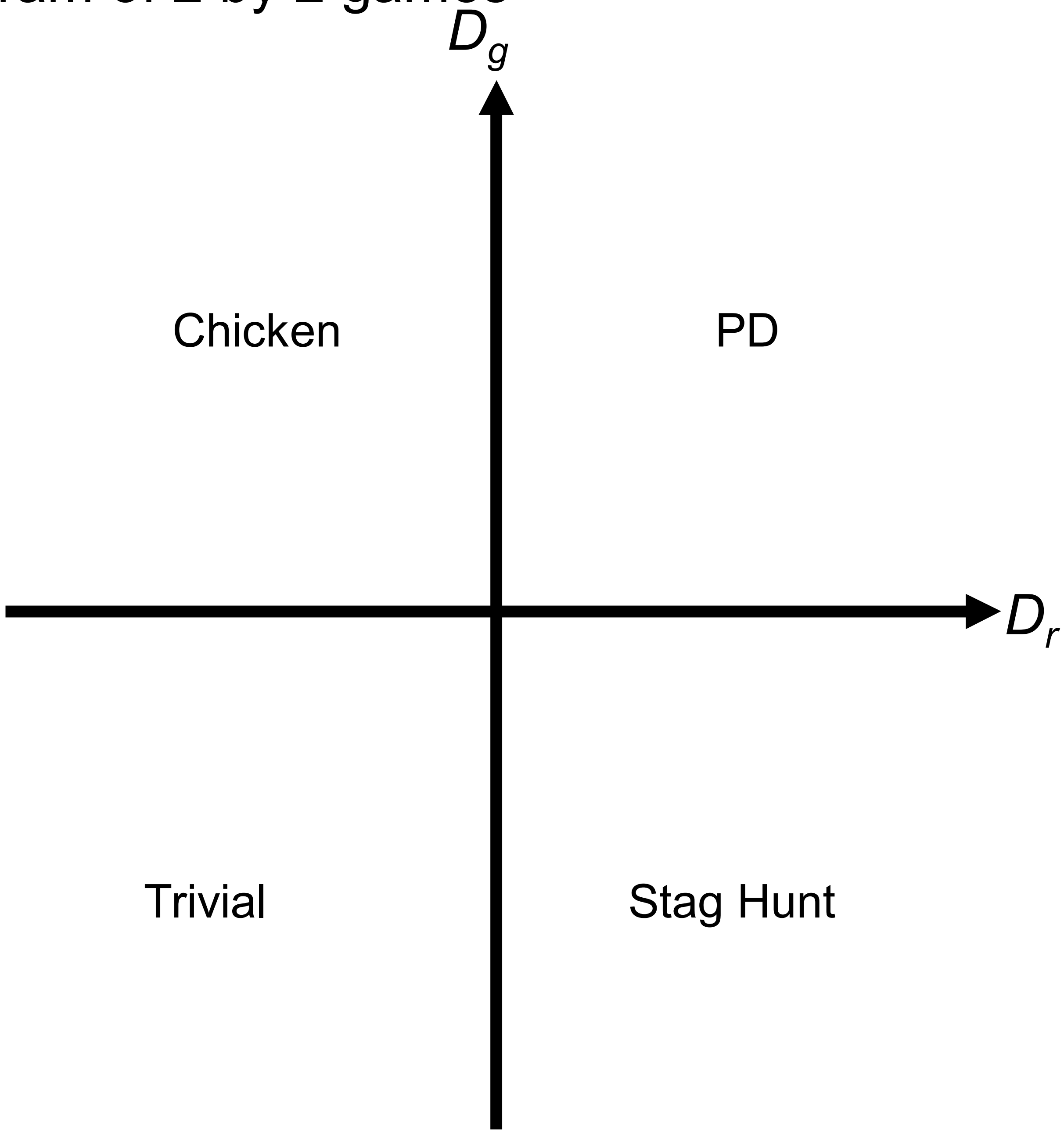
$$P < S \wedge R < T \Leftrightarrow P - S = D_r < 0 \wedge T - R = D_g > 0.$$

Summing up all so far, we obtain;

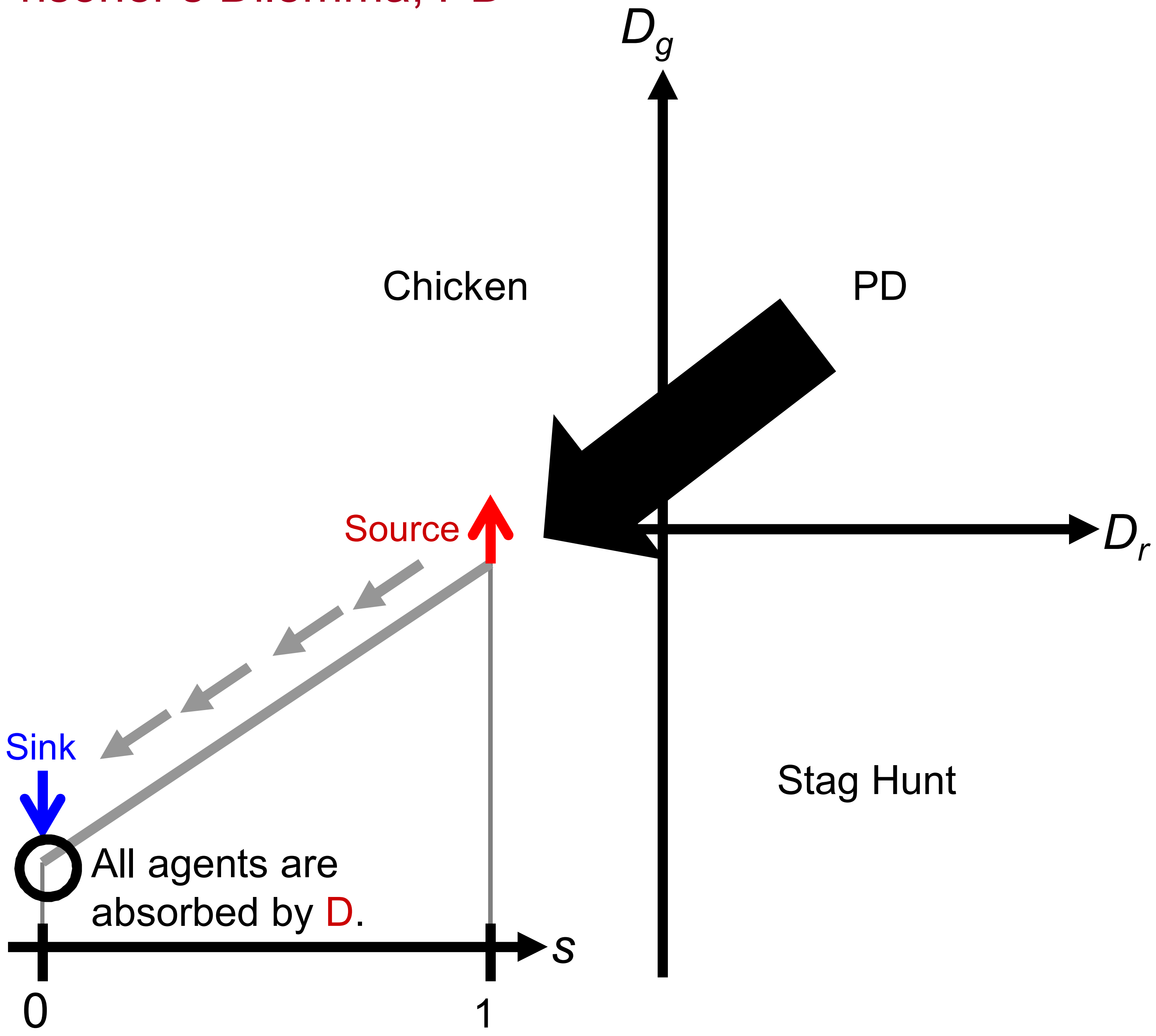
Game class	Trait	Nash Equilibrium	Sing of GID; D_g	Sing of RSD; D_r	Source or sink at Equilibrium; s^*		
					$(1,0)$	$(0,1)$	$\left(\frac{D_r}{D_g - D_r} \quad \frac{-D_g}{D_r - D_g}\right)$
PD	**D**-dominate	$(0,1)$	+	+	Source	sink	Saddle
Chicken	Polymorphic	$\left(\frac{D_r}{D_g - D_r} \quad \frac{-D_g}{D_r - D_g}\right)$	+	-	Source	Source	Sink
Stag Hunt	Bi-stable	$(0,1)$ or $(1,0)$	-	+	Sink	Sink	Source
Trivial	**C**-Dominate	$(1,0)$	-	-	Sink	Source	Saddle

Where $$s^* = \left(\frac{P-S}{P-T-S+R} \quad \frac{R-T}{P-T-S+R}\right) = \left(\frac{D_r}{D_g - D_r} \quad \frac{-D_g}{D_r - D_g}\right)$$

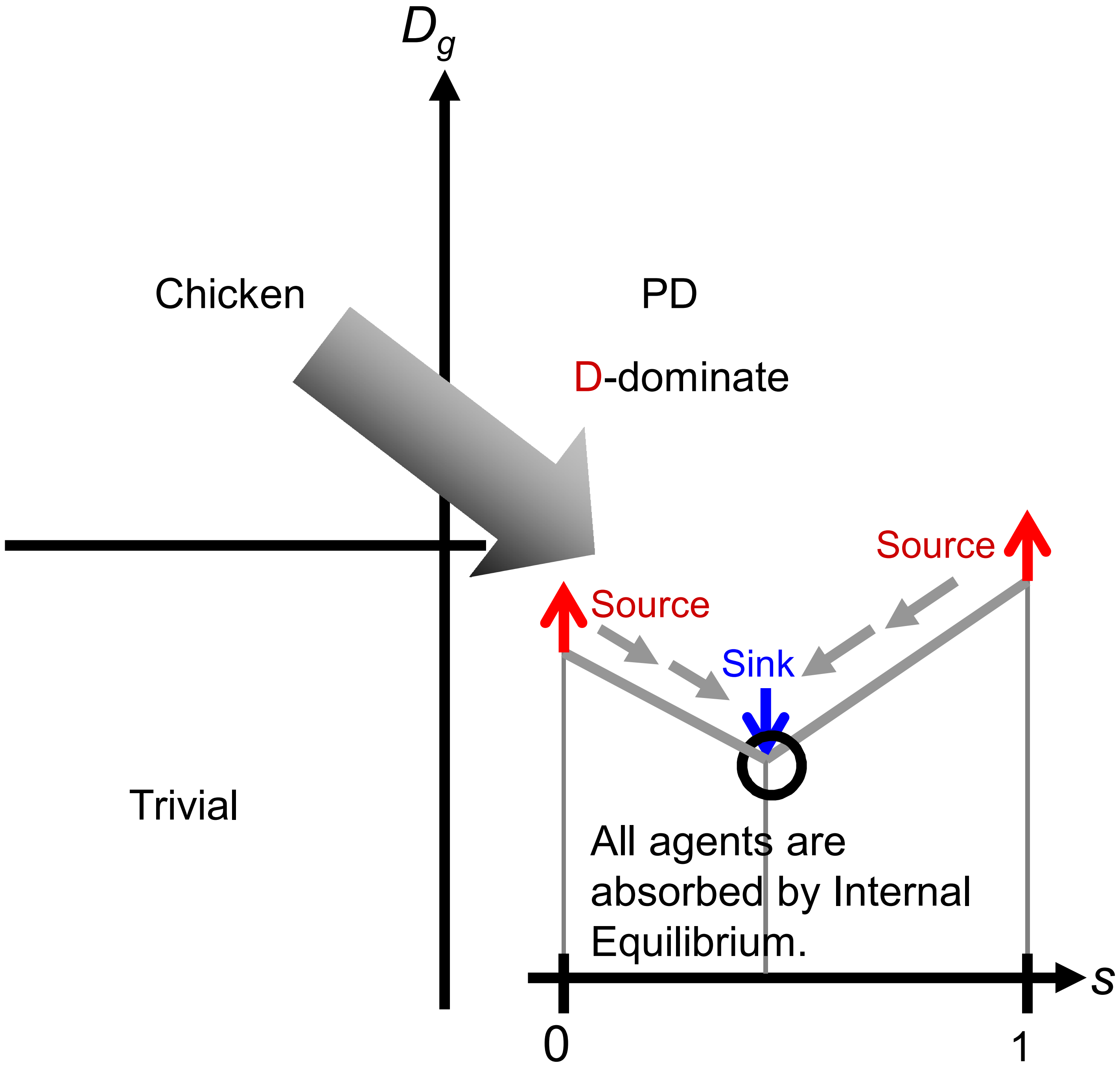
Phase diagram of 2 by 2 games



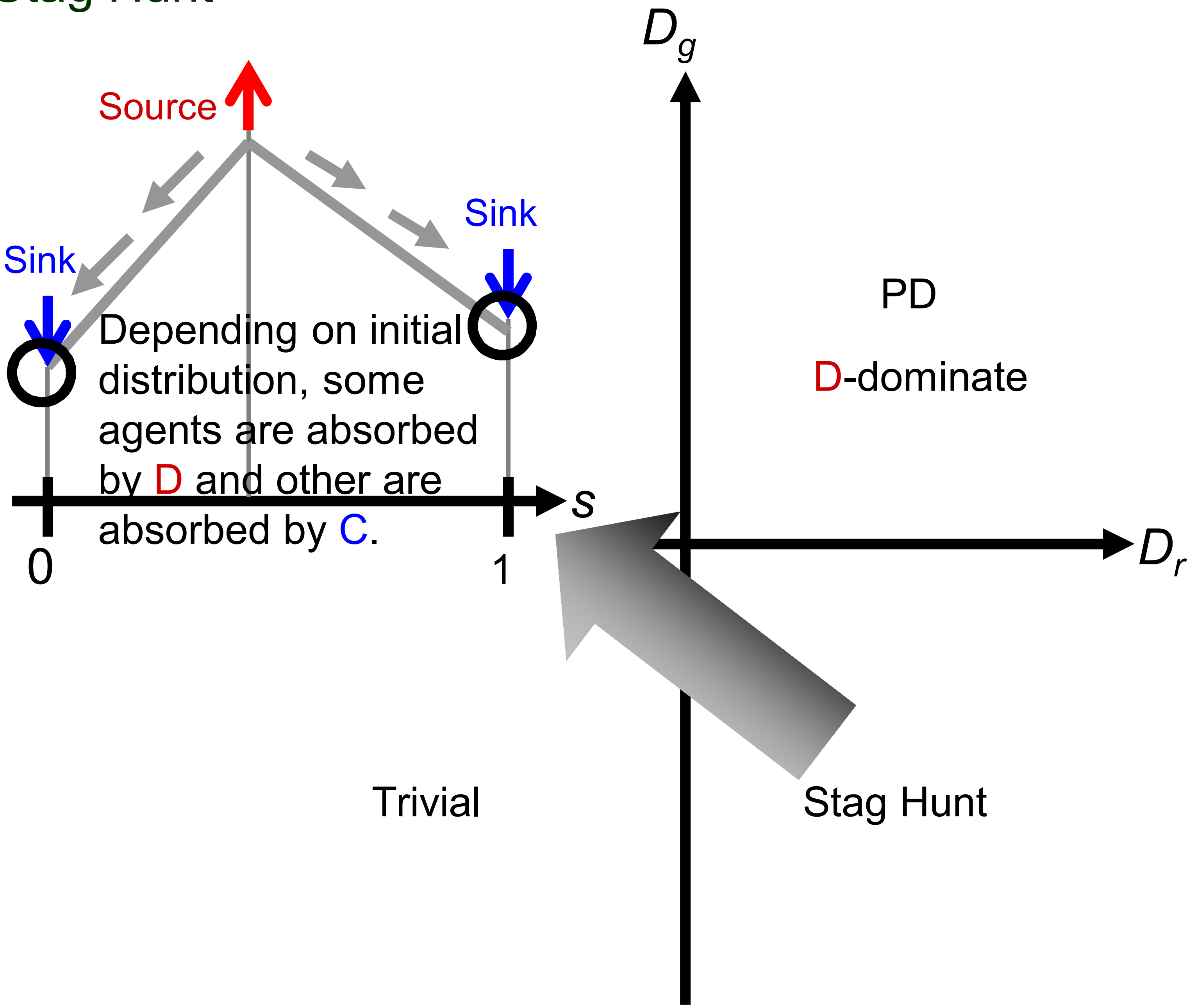
Prisoner's Dilemma, PD



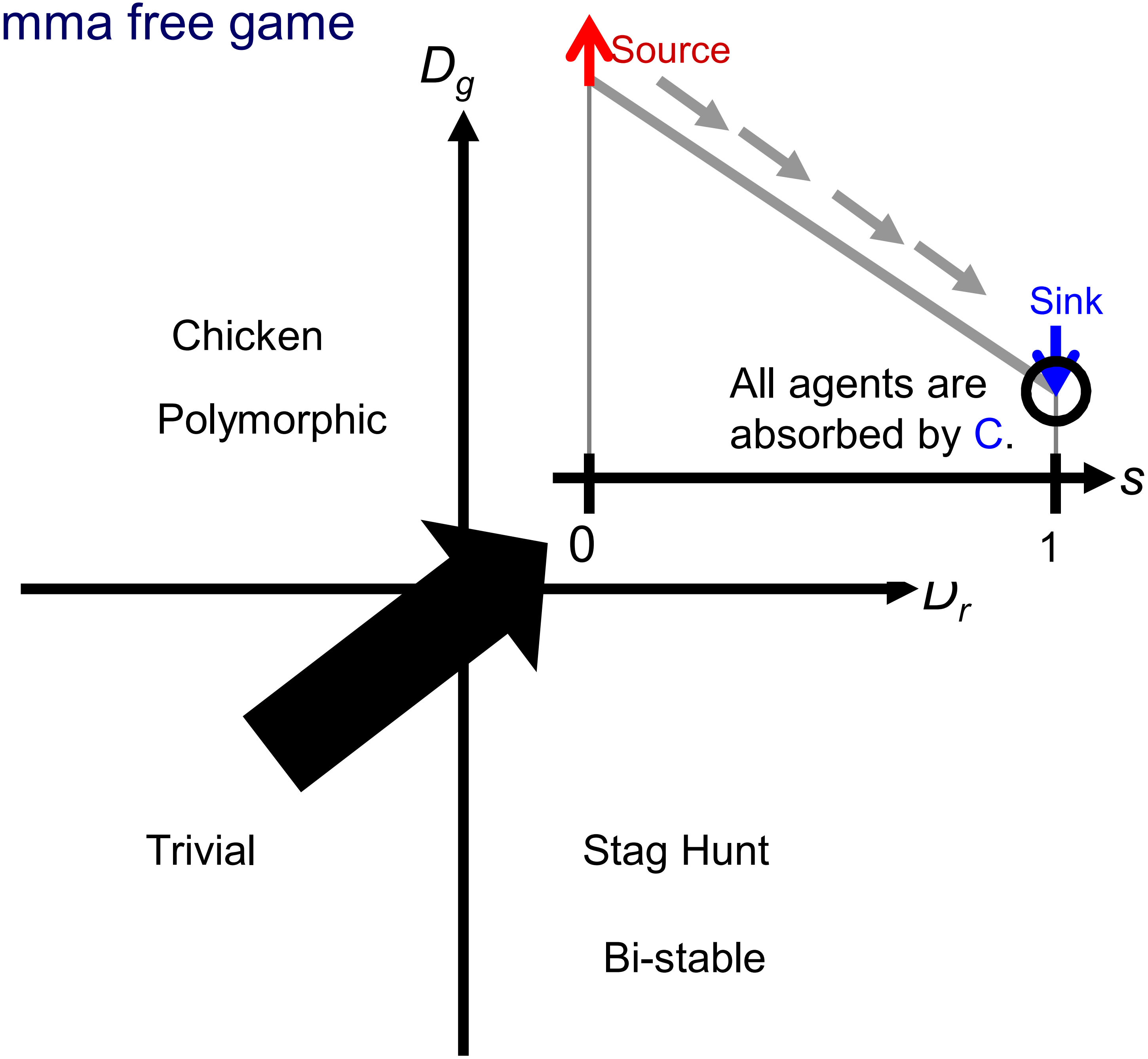
Chicken


D_g
Chicken
PD
D-dominate
Trivial
Source
Source
Sink
All agents are absorbed by Internal Equilibrium.
0
1
s

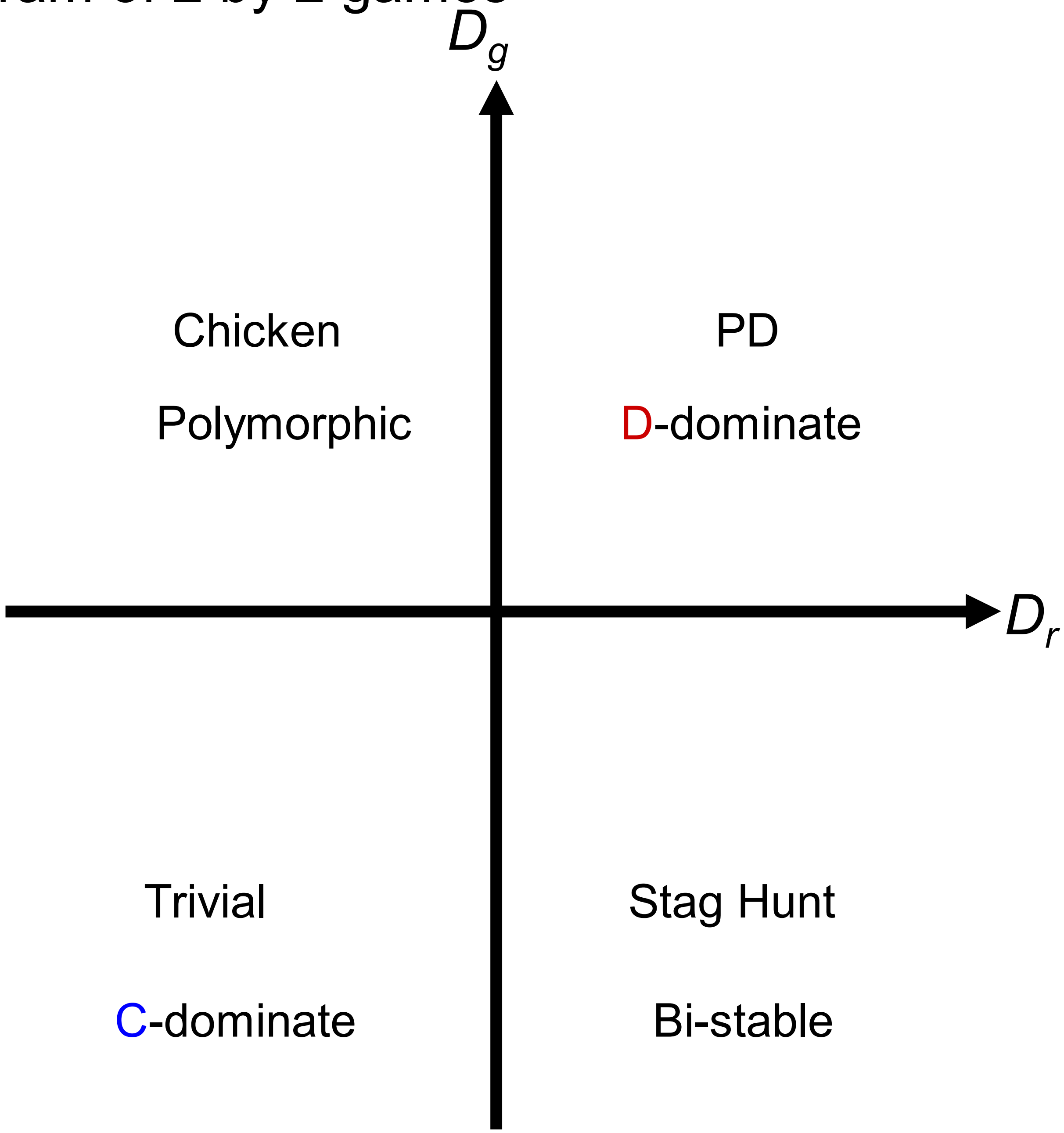
Stag Hunt


Source
Sink
Sink
Depending on initial distribution, some agents are absorbed by D and other are absorbed by C.
0
1
s
D_g
D_r
PD
D-dominate
Trivial
Stag Hunt

Trivial, dilemma free game



Phase diagram of 2 by 2 games



Backgrounds & Purpose

Most previous studies

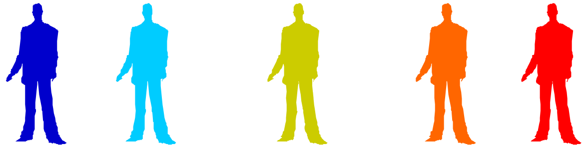
Entirely cooperation ⟺ Entirely defection

Agents can offer either cooperation (C) or defection (D)

Discrete strategy

The real world

Entirely cooperation ⟺ Entirely defection



Actual options might be continuous rather than discrete

Continuous or mixed strategy

One crucial question is whether there is a considerable difference in game equilibria between the continuous or mixed strategies and those of discrete strategies?

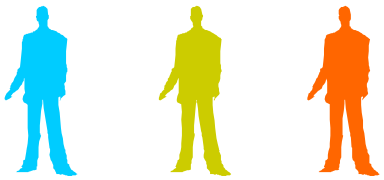
Setting for continuous, and mixed strategy games

Continuous strategy

1. Strategy value: $s_i \in [0,1]$

$s_i=1$ complete cooperation

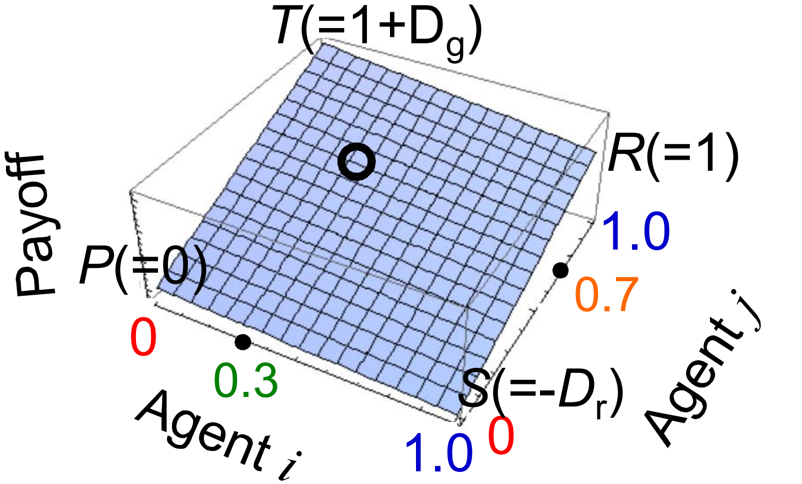
$s_i=0$ complete defection



(0.8) (0.5) (0.2)

2. Payoff function

$$\pi(s_i, s_j) \equiv -D_r s_i + (1 + D_g) s_j + (-D_g + D_r) s_i s_j$$

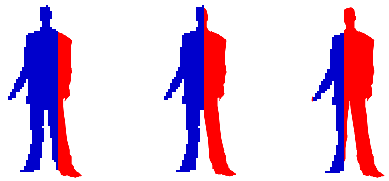


Mixed strategy

1. Strategy value: $s_i \in [0,1]$

$s_i=1$ complete cooperation

$s_i=0$ complete defection



(0.8) (0.5) (0.2)

Agents can only offer either C or D according to this strategy

C when Rnd[] < s_i, otherwise *D*

Rnd[ ]: a random number

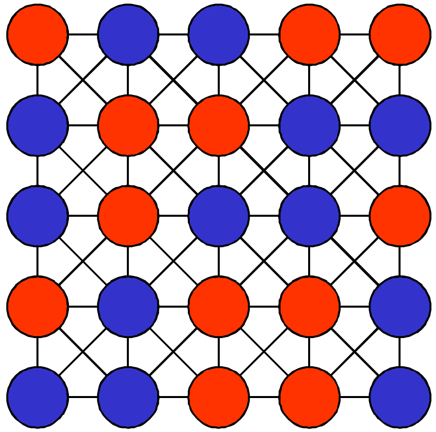
2. Payoff function

Agent *i* \ Agent *j*	C	D
C	1, 1	$-D_r$, $1+D_g$
D	$1+D_g$, $-D_r$	0, 0

Results

Averaged cooperation fraction

Games are played on lattices (k = 8)



	C	D
C	1	$-D_r$
D	$1+D_g$	0

D_g; GID D_r; RAD