Special lecture series of
Environment Energy Engineering

Environmental problems can be likened to social dilemma games.

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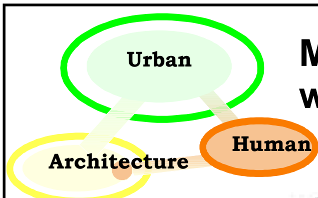
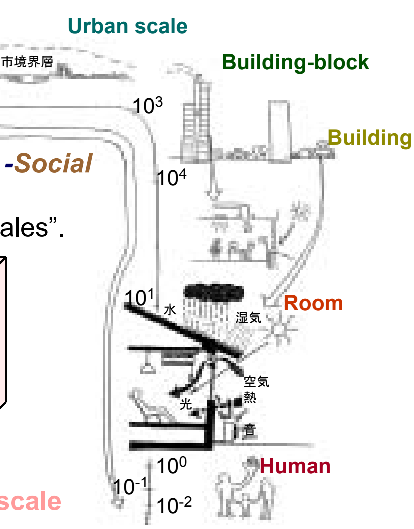
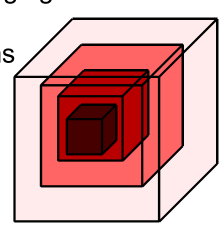


Mutually-interpenetrative view over wide spatial-scales

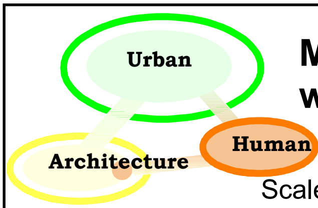


To elaborate the ***Human*** **-** ***Environment*** **-*Social System***, it's important a concept of "Simultaneous" or "Bridging to various scales".

Two physical systems having neighboring special scales are mutually connected through boundary conditions.



Small scale⟵Interaction⟶**Large scale**

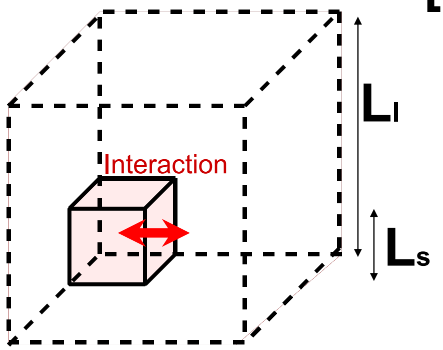


Mutually-interpenetrative view over wide spatial-scales

Scale velocity; $\dfrac{[L]}{[T]}$

Because of sharing the interaction through the boundary, the scale velocity MUST be also shared.

$$\frac{[L_l]}{[T_l]} \fallingdotseq \frac{[L_s]}{[T_s]}$$

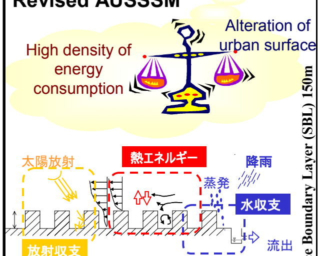


When one deals with both Large- and Small-systems simultaneously, a numerical solution scheme requires $[T_s] = [T_l]$.

Grid size for the Large-scale system is consistent with that of the Small-scale system for sure $[L_s] \fallingdotseq [L_l]$. → Huge computational resource is requisite.

改良・建築-都市-土壌連成系モデル

Revised Architecture-Urban-Soil Simultaneous Simulation Model, **Revised AUSSSM**



Urban atmosphere sub-model

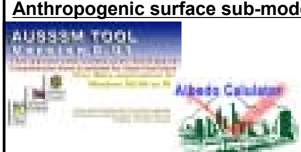
0 Eq. model, Gambo's turbulent length scale

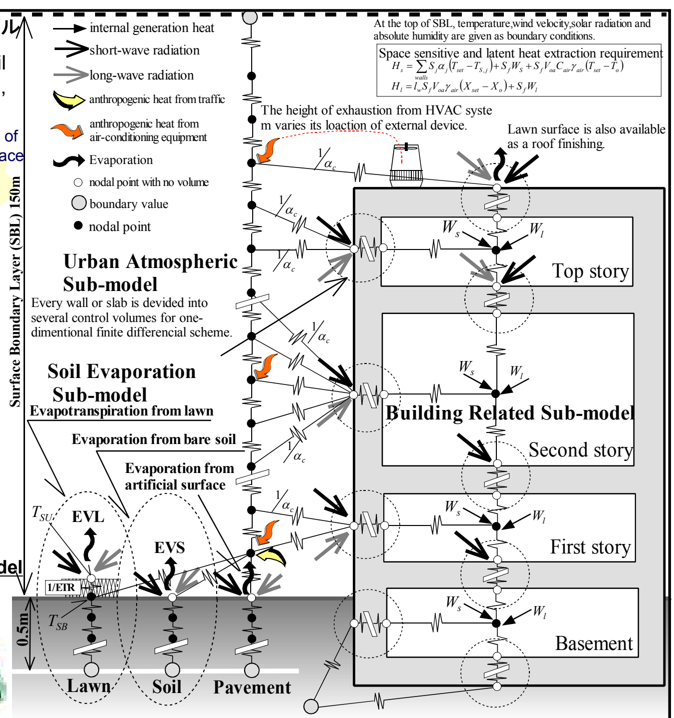
Building thermal system sub-model

Rooms, HVAC systems etc

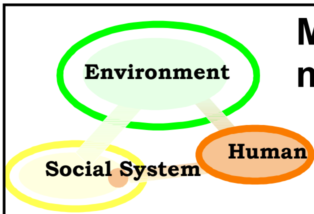
Soil, vegetation sub-model

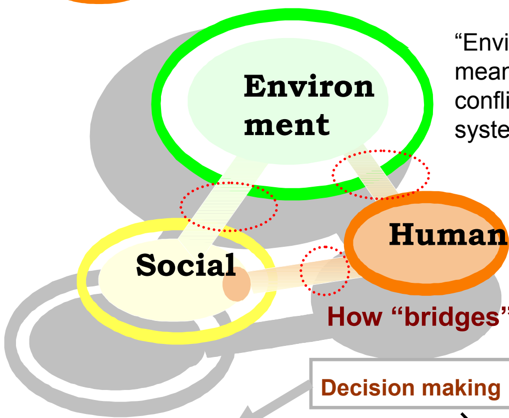
Anthropogenic surface sub-model





Mutually-interpenetrative view over mutually different systems

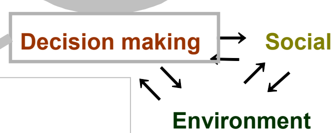




"Environmental problems" mean social dilemmas conflicting those three systems.

n.

How "bridges" are defined?



Science for complex system

Evolutionary game theory, Multi-agent simulation, Artificial intelligence (GA, NNw etc)

What is the *Game Theory* ?

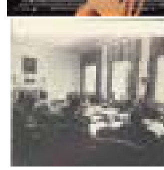
Game theory is a study of strategic decision making. More formally, it is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers."

John von Neumann & Oskar Morgenstern; Theory of games and economic behavior, 1944.

Game theory has been widely recognized as an important tool in many fields; economics, political science, psychology, as well as biology, information science and even statistical physics. Eight game-theorists, including John Nash have won the Nobel Memorial Prize in Economic Sciences, and John Maynard Smith was awarded the Crafoord Prize for his application of game theory to biology.

Zero-sum (Constant-sum) games

(Japanese) Chess, Go. Minimax theorem (von Neumann); For every two-person, zero-sum game with finitely many strategies, there exists a value V and a mixed strategy for each player, such that (a) Given player 2's strategy, the best payoff possible for player 1 is V, and (b) Given player 1's strategy, the best payoff possible for player 2 is −V.



Non zero-sum (Non constant-sum) games

Many applications happening in real world. Social dilemma, Prisoner's Dilemma, Chicken games etc.

Cuba Crisis -->Chicken game?

2 by 2 game

Agent1 Agent2


	Player 2 chooses *Left*	Player 2 chooses *Right*
Player 1 chooses *Up*	4, 3	−1, −1
Player 1 chooses *Down*	0, 0	3, 4

Normal form or payoff matrix of a 2-player, 2-strategy game

Agent2 \ Agent1	Cooperation (C)	Defection (D)
Cooperation (C)	R, R	S, T
Defection (D)	T, S	P, P

R; Reward, T; Temptation, S; Sucker, P; Punishment

Dynamics in nonlinear systems

Nonlinear equation

$$\frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

A question, which seems crucially important to see basic feature of the system, is whether the system has steady states (equilibriums) or not.

If so, how are those?

If the answer for this question can be drawn through analytical way, that's much better than any numerical approaches.

Analytical approach concerning equilibrium (steady-state) for Linear systems

$$\frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

For simplicity, we disregarded impacts resulting from boundary conditions, which makes sure only to be concerned on the system body.

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\,\mathbf{x} \Leftrightarrow \frac{1}{\mathbf{x}}d\mathbf{x} = \mathbf{A}dt \Leftrightarrow \mathbf{x} = \exp[\mathbf{A}\,t] + \mathbf{c}$$

Equilibrium ⇔ Steady-state

In this case,

$$\Leftrightarrow \frac{d\mathbf{x}}{dt} = \mathbf{0} \Leftrightarrow \dot{\mathbf{x}} = \mathbf{0} \Rightarrow \mathbf{A}\mathbf{x}^* = \mathbf{0} \Leftrightarrow \mathbf{x}^* = \mathbf{0}$$

Equilibrium ⇔ Steady-state $\Leftrightarrow \frac{d\mathbf{x}}{dt} = \mathbf{0} \Leftrightarrow \dot{\mathbf{x}} = \mathbf{0}$

Suppose $t \to \infty$.

Only when $\mathbf{x}(t) = exp[\mathbf{A}\,t] \to \mathbf{0}$,

this system has Stable Equilibrium (steady-state).

Scalar space

If $a<0$ then $\exp[a\,t] \to 0$.

Vector-Matrix space

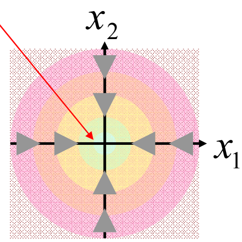
If all eigen values of $\mathbf{A}$ (there are n eigen values if $\mathbf{A}$ is defined as n-square matrix) are negative, $\exp[\mathbf{A}\,t] \to \mathbf{0}$.

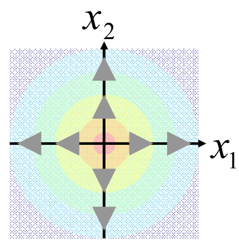
Thus, what we should investigate is whether signs of all eigen values of $\mathbf{A}$ are + or not.

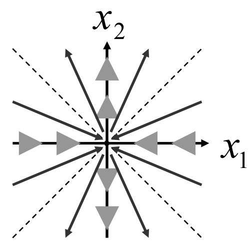
Equilibrium

$\mathbf{x}^*$

Stable	Unstable	Unstable
Sink	Source	Saddle

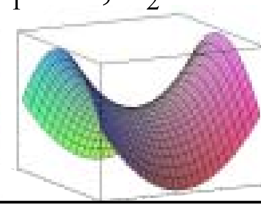






$\lambda_1, \lambda_2 < 0$ $\quad$ $\lambda_1, \lambda_2 > 0$ $\quad$ $\lambda_1 < 0, \lambda_2 > 0$

Eigen values of $\mathbf{A}$



$$\frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

Time-continuous system

Time discretization by Forward FDM

$$\mathbf{x_{k+1}} - \mathbf{x_k} = \Delta t \cdot \mathbf{A}\,\mathbf{x_k}$$

$$\Leftrightarrow \mathbf{x_{k+1}} = \left(\Delta t \cdot \mathbf{A} + \mathbf{E}\right)\mathbf{x_k}$$

Linear mapping

Here, let us remind the Stability condition of Transition Matrix; **T** in System-state Equation.

The necessary and sufficient condition for convergence is;

$$\left|\mathrm{Max}\left[\mathrm{eigen}\left[\mathbf{T}\right]\right]\right| \leq 1$$

$$\mathbf{x}_{\mathbf{k}+1} = \underbrace{(\Delta t \cdot \mathbf{A} + \mathbf{E})}_{\mathbf{T}} \mathbf{x}_{\mathbf{k}}$$

Now, let us assume that the system instinctively stable; e.g.;

$$\mathrm{Max}[\mathrm{eigen}[\mathbf{A}]] \leq 0 \quad .$$

We know; $\mathrm{eigen}[\mathbf{E}] = 1$.

It is worthwhile to note that even though an instinctive system is stable, its mapping system may be unstable, because the following situation might happen;

$$\mathrm{Max}[\mathrm{eigen}[\mathbf{T}]] < -1 \quad .$$

It is remarkably amazing that a mapping operation by time-Forward FDM may cause unstable (numerical divergence) even though the system instinctively has stability.

Let us take a look when time-Backward FDM is applied.

$$\frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

Time discretization by Backward FDM

$$\mathbf{x}_{\mathbf{k}+1} - \mathbf{x}_{\mathbf{k}} = \Delta t \cdot \mathbf{A}\, \mathbf{x}_{\mathbf{k}+1}$$

$$\Leftrightarrow \mathbf{x}_{\mathbf{k}+1} = [1 - \Delta t \cdot \mathbf{A}\ ]^{-1} \mathbf{x}_{\mathbf{k}} = \mathbf{T}\mathbf{x}_{\mathbf{k}}$$

If an instinctive system is stable, its mapping system is always stable, because;

$$0 < \mathrm{Max}[\mathrm{eigen}[\mathbf{T}]] < 1 \quad .$$

It is also notable that a mapping operation by time-Backward FDM is always consistent with the system instinctive stability.

Thus, if a system is instinctively stable, its mapping by Backward FDM is stable as well.

Analytical approach concerning equilibrium (steady-state) for Nonlinear systems

Pseudo (quasi)-linearization approach should be applied.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

Let us take the Taylor development of nonlinear function f around an equilibrium $\mathbf{x}=\mathbf{x}^*$.

$$\mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{x}^*) + \mathbf{f}'(\mathbf{x}^*)(\mathbf{x}-\mathbf{x}^*) + \frac{\mathbf{f}''(\mathbf{x}^*)}{2!}(\mathbf{x}-\mathbf{x}^*)^2 + \cdots$$

$$\Leftrightarrow \mathbf{f}(\mathbf{x}) \cong \underline{\mathbf{f}(\mathbf{x}^*)} + \mathbf{f}'(\mathbf{x}^*)(\mathbf{x}-\mathbf{x}^*)$$

=0; because of the definition of equilibrium

$$\mathbf{f}(\mathbf{x}) = \mathbf{f}'(\mathbf{x}^*)(\mathbf{x}-\mathbf{x}^*)$$

$$\mathbf{f}(\mathbf{x}) = \mathbf{f}'(\mathbf{x}^*)(\mathbf{x}-\mathbf{x}^*) = \underline{\mathbf{f}'(\mathbf{x}^*)}\,\underline{\mathbf{x}} - \underline{\mathbf{f}'(\mathbf{x}^*)\mathbf{x}^*}$$

Matrix consisted of constant values. Unknown vector. Vector consisted of constant values.

Now, nonlinear function f has been approximated by a linear function like;

$$\underline{\mathbf{A}}\,\underline{\mathbf{x}} + \underline{\text{Constant}}.$$

To the end, we can say that;

whether the Equilibrium, $\mathbf{x}=\mathbf{x}^*$, of $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ can be evaluated by eigen values of;

$$\mathbf{f}'(\mathbf{x}^*) = \left.\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*} = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_n(\mathbf{x})}{\partial x_n} \end{bmatrix}_{\mathbf{x}=\mathbf{x}^*}$$

Jacobi Matrix

Thus,

if all eigen values of Jacobi Matrix are negative, the equilibrium $\mathbf{x}=\mathbf{x}^*$ is stable sink point.

if all eigen values of Jacobi Matrix are positive, the equilibrium $\mathbf{x}=\mathbf{x}^*$ is unstable source point.

If both negative and positive values are co-exist, the equilibrium $\mathbf{x}=\mathbf{x}^*$ is unstable saddle point.

Application; Analytical approach concerning equilibrium (steady-state) for Nonlinear systems

2-player 2-strategy game (2 by 2 game)

Class	Dilemma?	GID	RAD
Prisoner's Dilemma; PD	Yes	Yes	Yes
Chicken (Snow Drift; Hawk-Dove)	Yes	Yes	No
Stag Hunt; SH	Yes	No	Yes
Trivial	No	No	No

Basic Assumption

- Infinite population.
- One-shot game; well-mixed situation (with neither social viscosity nor assortment among agents).

Prisoner's Dilemma

Agent1 Agent2

Agent2 / Agent1	Cooperation (C)	Defection (D)
Cooperation (C)	*R*, *R*	*S*, *T*
Defection (D)	*T*, *S*	*P*, *P*

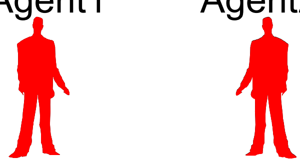
R; Reward, *T*; Temptation, *S*; Sucker, *P*; Punishment

Prisoner's Dilemma

	C	D
C	R, R	S, T
D	T, S	P, P

R; Reward, *T*; Temptation
S; Sucker, *P*; Punishment

Agent1 Agent2



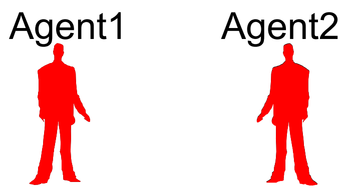
$2R(=8) > T+S(=6) > 2P(=4)$

Agent2 / Agent1	Cooperation (C)	Defection (D)
Cooperation (C)	5, 5 Equal Pareto Optimum	1, 7
Defection (D)	7, 1	3, 3 Nash Equilibrium

Gamble-Intending Dilemma (GID); $D_g = T - R = 7 - 5 > 0$

Risk-Averting Dilemma (RAD); $D_r = P - S = 3 - 1 > 0$

Prisoner's Dilemma



	C	D
C	R	S
D	T	P

R; Reward, *T*; Temptation
S; Sucker, *P*; Punishment

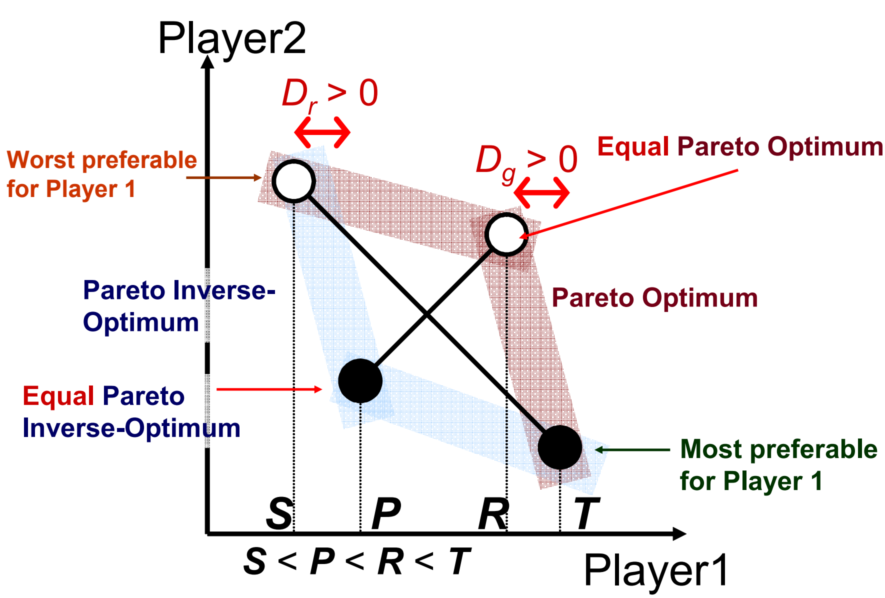
$2R(=8) > T+S(=6) > 2P(=4)$

Agent2 / Agent1	Cooperation (C)	Defection (D)
Cooperation (C)	5 Equal Pareto Optimum	1
Defection (D)	7	3 Nash Equilibrium

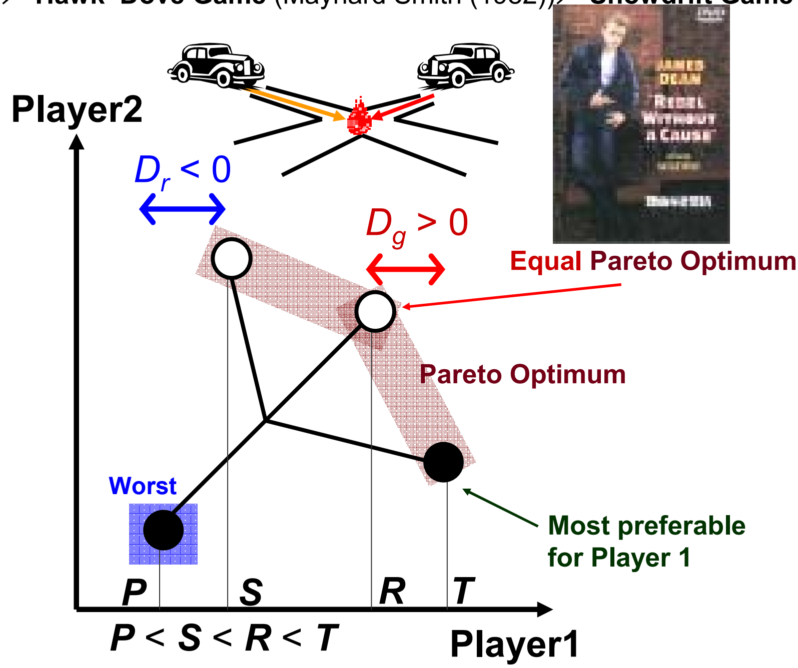
Gamble-Intending Dilemma (GID); $D_g = T - R = 7 - 5 > 0$

Risk-Averting Dilemma (RAD); $D_r = P - S = 3 - 1 > 0$

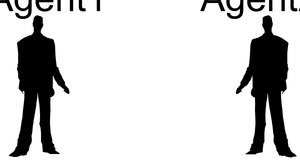
Prisoner's Dilemma



Chicken ⁄ Hawk–Dove Game (Maynard Smith (1982)) ⁄ Snowdrift Game



Chicken



	C	D
C	R	S
D	T	P

R; Reward, *T*; Temptation
S; Sucker, *P*; Punishment

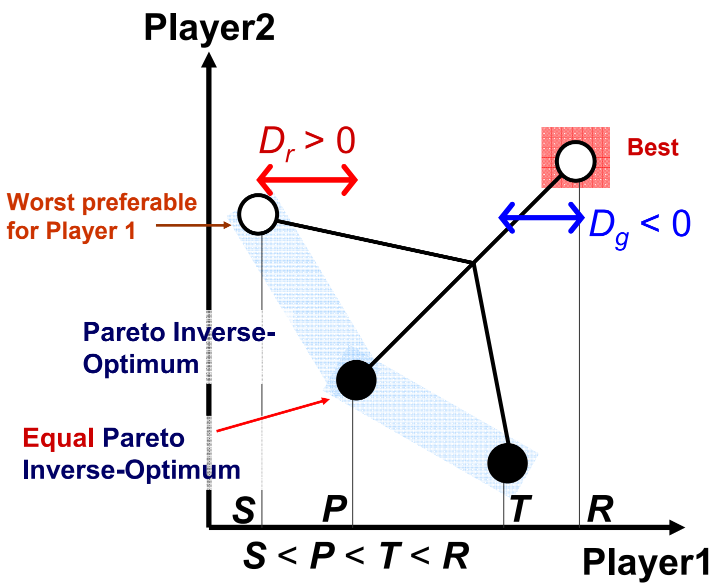
$2R(=8) > T+S(=6) > 2P(=4)$

Agent2 / Agent1	Cooperation (C)	Defection (D)
Cooperation (C)	5 Equal Pareto Optimum	3 Nash Equilibrium
Defection (D)	7 Nash Equilibrium	1 Worst

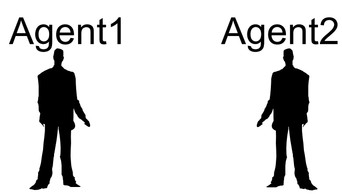
Gamble-Intending Dilemma (GID); $D_g = T - R = 7 - 5 > 0$

Risk-Averting Dilemma (RAD); $D_r = P - S = 3 - 1 < 0$

Stag Hunt／ Inspired by Jean-Jacques Rousseau; “Discours sur l'origine et les fondements de l'inégalité parmi les hommes” (Chapter 2)



Stag Hunt



	C	D
C	R	S
D	T	P

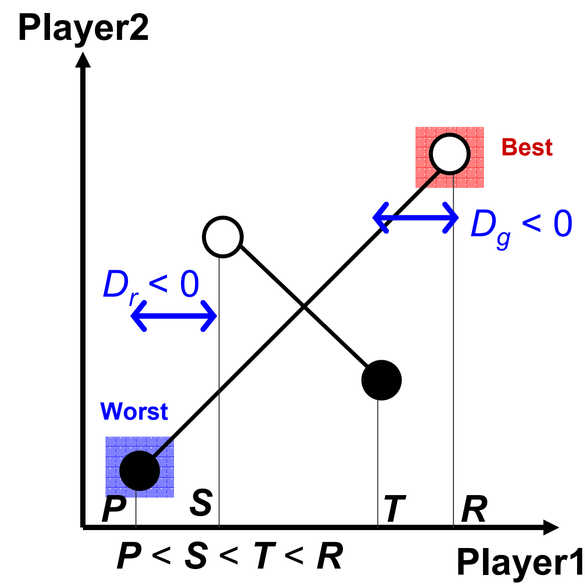
R; *R*eward, *T*; *T*emptation
S; *S*ucker, *P*; *P*unishment

Agent2 / Agent1	Cooperation (C)	Defection (D)
Cooperation (C)	Nash Equilibrium 7 Best=Equal Pareto Optimum	1
Defection (D)	5	3 Nash Equilibrium

Gamble-Intending Dilemma (GID); $D_g=T-R=5-7<0$

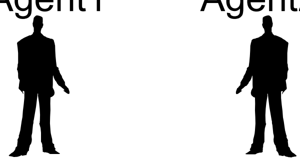
Risk-Averting Dilemma (RAD); $D_r=P-S=3-1>0$

Trivial Dilemma Free game



Trivial

Agent1 Agent2



	C	D
C	R	S
D	T	P

R; Reward, *T*; Temptation
S; Sucker, *P*; Punishment

Agent2 / Agent1	Cooperation (C)	Defection (D)
Cooperation (C)	7 (Nash Equilibrium; Best=Equal Pareto Optimum)	3
Defection (D)	5	1

Gamble-Intending Dilemma (GID); $D_g = T - R = 5 - 7 < 0$

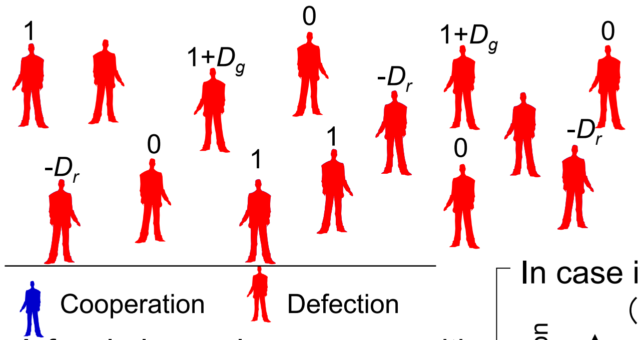
Risk-Averting Dilemma (RAD); $D_r = P - S = 1 - 3 < 0$

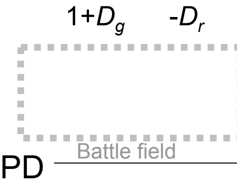
Evolutionary game

2 by 2 game considered time evolution

	C	D
C	1	$-D_r$
D	$1+D_g$	0

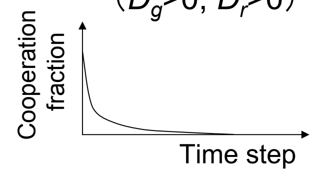
D_g; GID
D_r; RAD





1. A focal player plays a game with a randomly selected opponent.
2. Strategy (whether C or D) adaptation based on obtained payoff is considered.

In case if PD (D_g>0, D_r>0)



You never see emerging cooperation, unless some additional mechanism for social viscosity is implemented.

What is *Social Viscosity*?

- Kin selection
- Direct reciprocity
- Indirect Reciprocity
- Network Reciprocity

- Group selection

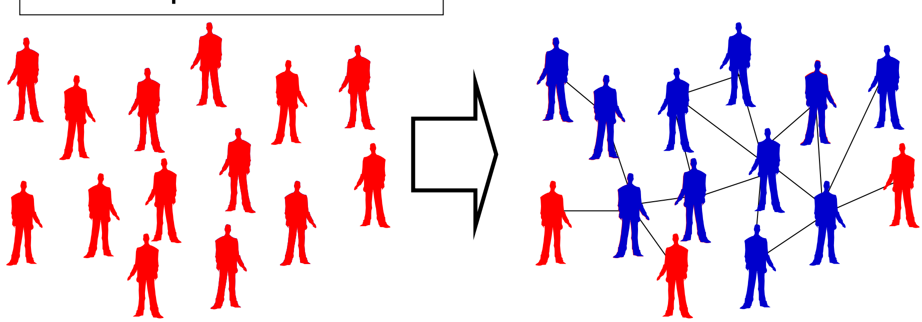
A restricted relation among agents

↓

Lessing *Anonymity*

↓

Emerging cooperation



Well-mixed situation　　A Game on a network

Let us back to the Basic Assumption again;

- Infinite population.
- One-shot game; well-mixed situation (with neither social viscosity nor assortment among agents).

Let us describe Cooperation and defection strategies by;

$$^{T}\mathbf{e_1} = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad ; \text{C}$$

$$^{T}\mathbf{e_2} = \begin{pmatrix} 0 & 1 \end{pmatrix} \quad ; \text{D}$$

Also, let us define game structure, i.e. payoff matrix as below;

$$\begin{bmatrix} R & S \\ T & P \end{bmatrix} \equiv \mathbf{M}$$

Further, let us define strategy frequency among agents at a certain time step as below; $^{T}\mathbf{s} = \begin{pmatrix} s_1 & s_2 \end{pmatrix}$

Fraction of C D

By simplex constraint; $s_2 = 1 - s_1$.

Let us think a simple example. When a focal player who offers D, how much of payoff expectation she can get in case of paying with another D player as her game opponent?

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \cdot \begin{bmatrix} P & S \\ T & P \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = P$$

By analogy, payoff expectations of both a C and D players respectively paying with average players at this time step are;

$$^{T}\mathbf{e_1} \cdot \mathbf{M}\,\mathbf{s}$$

$$^{T}\mathbf{e}_2 \cdot \mathbf{M}\,\mathbf{s}$$

Let us consider the following system dynamics, called **Replicator Dynamics**, which is thought to be a good model for describing the reproduction process of population dynamics for animal species.

$$\frac{\dot{s}_i}{s_i} = {}^T\mathbf{e_i}\cdot\mathbf{M}\,\mathbf{s} - {}^T\mathbf{s}\cdot\mathbf{M}\,\mathbf{s}$$

Changing rate of strategy *i*; C when *i*=1 & D when *i*=2

Payoff expectations of a strategy *i* player paying with an average player at this time step

Payoff expectations of an average player paying with an average player at this time step

Implying benefit brought to a player who adopts strategy *i*.

Replicator Dynamics: $\frac{\dot{s}_i}{s_i} = {}^T\mathbf{e_i}\cdot\mathbf{M}\,\mathbf{s} - {}^T\mathbf{s}\cdot\mathbf{M}\,\mathbf{s}$ has three equilibriums.

Two obvious equilibriums are;

$(1,0)$; A state absorbed by C where all players offer C (C Dominate phase) .

$(0,1)$; A state absorbed by D where all players offer D (D Dominate phase) .

The third one is;

(Polymorphic phase).

A question is what dynamics would be if analytic approach is applied to the Replicator Dynamics, which is a (nonlinear) cubic equation for s_1 or s_2.

Let us describe Replicator Dynamics explicitly by substituting i=1 and 2.

$$\frac{\dot{s}_i}{s_i} = {}^T\mathbf{e_i} \cdot \mathbf{M}\,\mathbf{s} - {}^T\mathbf{s} \cdot \mathbf{M}\,\mathbf{s}$$

$$\Leftrightarrow \begin{cases} \dot{s}_1 = \left[(R-T)\cdot s_1 - (P-S)\cdot s_2\right]\cdot s_1 \cdot s_2 \\ \dot{s}_2 = -\left[(R-T)\cdot s_1 - (P-S)\cdot s_2\right]\cdot s_1 \cdot s_2 \end{cases}$$

When defining $\dot{s}_1 \equiv f_1(s_1, s_2)$ and $\dot{s}_2 \equiv f_2(s_1, s_2)$ as well as reminding Simplex constraint; $s_2 = 1 - s_1$, we know;

$$f_1 = -f_2$$

Again, Our current target is **to evaluate Eigen values of Jacobi Matrix at respective three equilibrium**; s^*.

$$\mathbf{f}'(\mathbf{x}^*) = \left.\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*} = \left.\begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_n(\mathbf{x})}{\partial x_n} \end{bmatrix}\right|_{\mathbf{x}=\mathbf{x}^*}$$

$$\begin{cases} \frac{\partial f_1}{\partial s_1} = -\frac{\partial f_2}{\partial s_1} = 3(-R+S+T-P)s_1^{\,2} \\ \qquad + 2(R-2S-T+2P)s_1 + S - P \\ \frac{\partial f_1}{\partial s_2} = -\frac{\partial f_2}{\partial s_2} = -3(-R+S+T-P)s_1^{\,2} \\ \qquad - 2(R-2S-T+2P)s_1 - S + P \end{cases}$$

We know two Eaigen values of $\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial s_1} & \frac{\partial f_1}{\partial s_2} \\ \frac{\partial f_2}{\partial s_1} & \frac{\partial f_2}{\partial s_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial s_1} & \frac{\partial f_1}{\partial s_2} \\ -\frac{\partial f_1}{\partial s_1} & -\frac{\partial f_1}{\partial s_2} \end{bmatrix}$ are;

0 and $\frac{\partial f_1}{\partial s_1} - \frac{\partial f_1}{\partial s_2}$ (its eiven vector is $(1,-1)$) .

Thus, what we should currently do is evaluate sings of $\lambda \equiv \frac{\partial f_1}{\partial s_1} - \frac{\partial f_1}{\partial s_2}$ at respective three equilibrium; s^*.

$$\lambda = \frac{\partial f_1}{\partial s_1} - \frac{\partial f_1}{\partial s_2} = 6(-R+S+T-P)s_1^{\,2} + 4(R-2S-T+2P)s_1 + 2(S-P)$$

(1) At $s^* = (1,0)$; $\lambda = -2R + 2T$.

Thus, for $\lambda < 0$, it must be $T - R = D_g < 0$.

(2) At $s^* = (0,1)$; $\lambda = 2S - 2P$.

Thus, for $\lambda < 0$, it must be $P - S = D_r > 0$.

(3) At $s^* = \left(\frac{P-S}{P-T-S+R} \quad \frac{R-T}{P-T-S+R} \right)$; $\lambda = 2\frac{(R-T)(P-S)}{R-S-T+P}$.

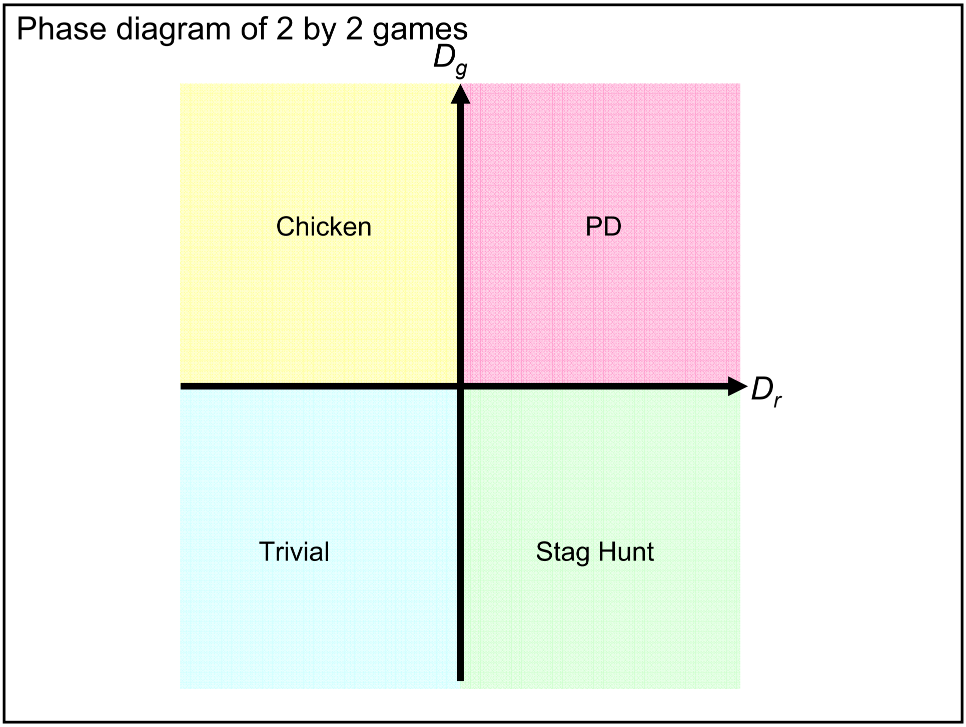
Thus, for $\lambda < 0$, it must be;

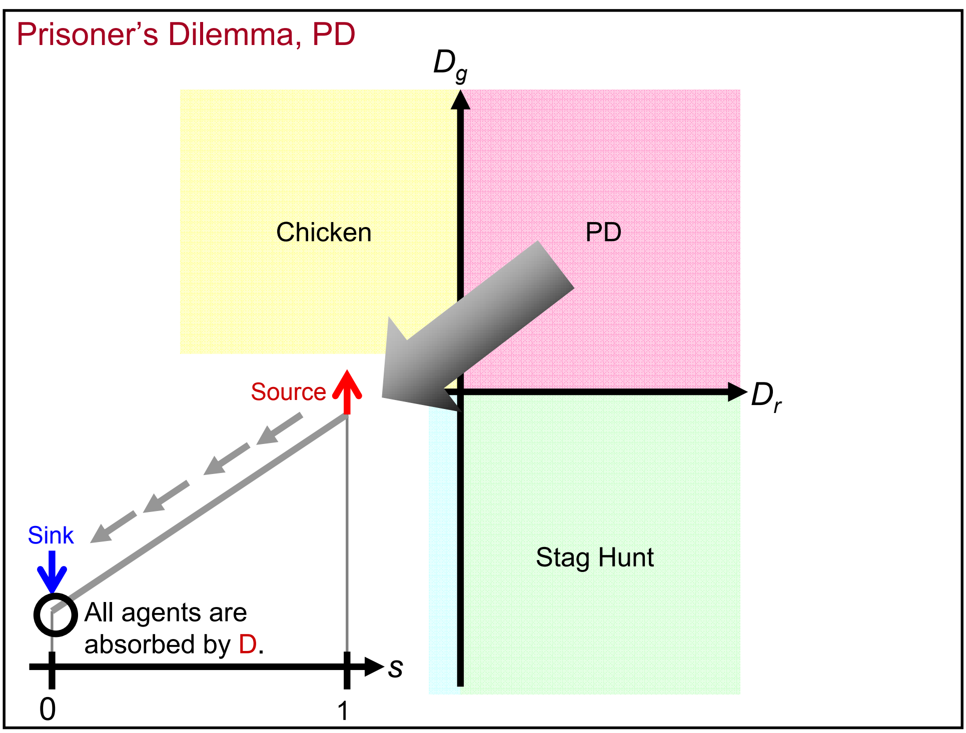
$$P < S \wedge R < T \Leftrightarrow P - S = D_r < 0 \wedge T - R = D_g > 0.$$

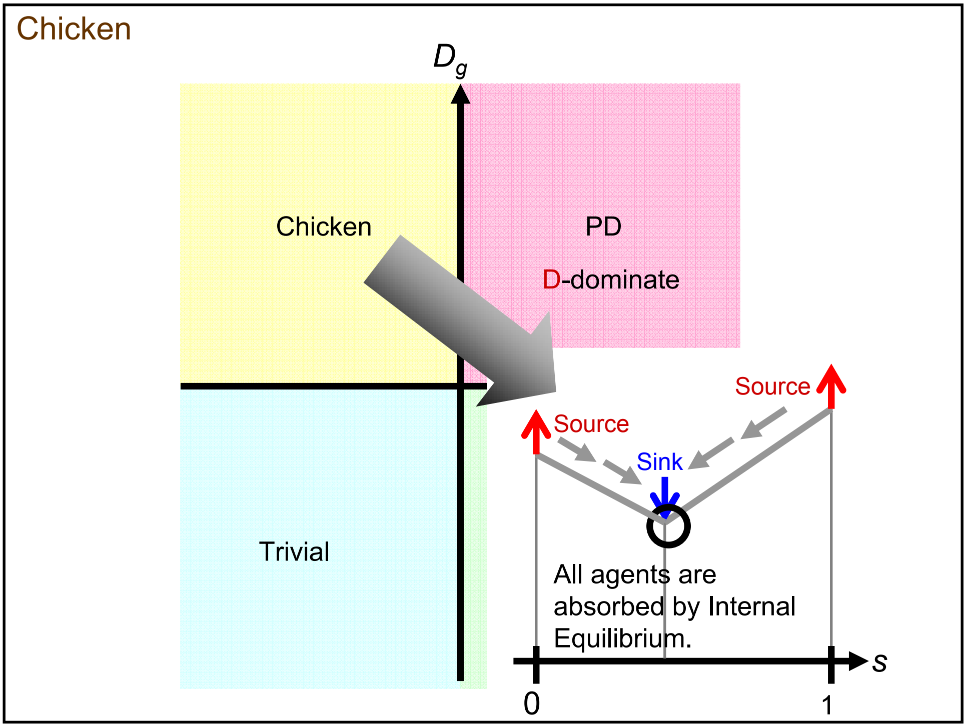
Summing up all so far, we obtain;

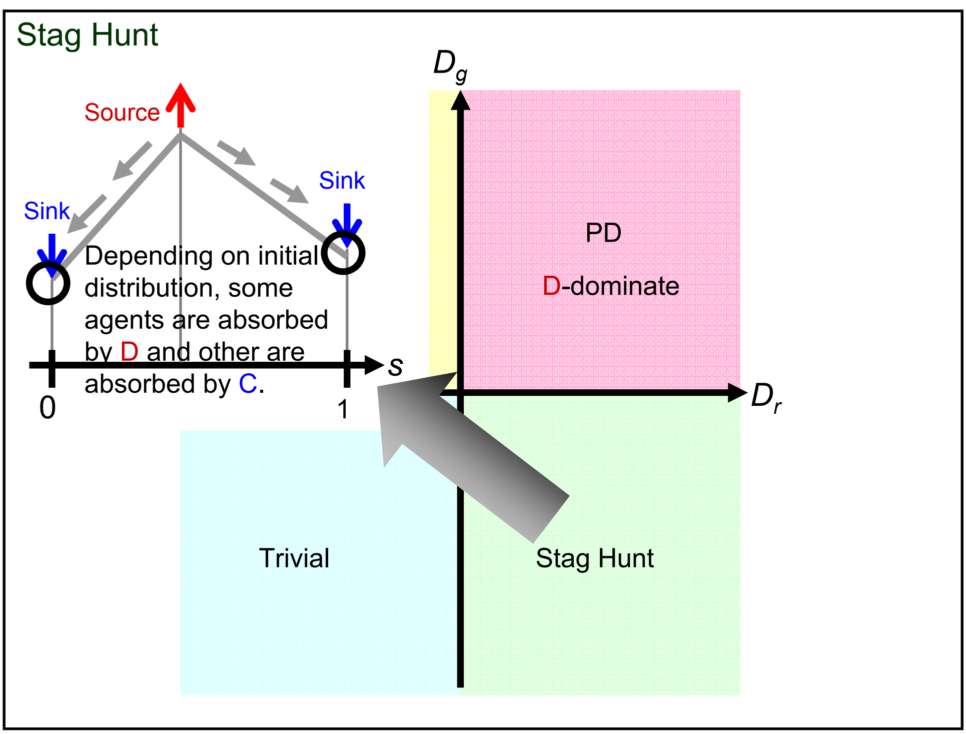
Game class	Trait	Nash Equilibrium	Sing of GID; D_g	Sing of RSD; D_r	Source or sink at Equilibrium; s^*		
					$(1,0)$	$(0,1)$	$\left(\frac{D_r}{D_g - D_r} \quad \frac{-D_g}{D_r - D_g} \right)$
PD	**D**-dominate	$(0,1)$	+	+	Source	sink	Saddle
Chicken	Polymorphic	$\left(\frac{D_r}{D_g - D_r} \quad \frac{-D_g}{D_r - D_g} \right)$	+	-	Source	Source	Sink
Stag Hunt	Bi-stable	$(0,1)$ or $(1,0)$	-	+	Sink	Sink	Source
Trivial	**C**-Dominate	$(1,0)$	-	-	Sink	Source	Saddle

Where $s^* = \left(\frac{P-S}{P-T-S+R} \quad \frac{R-T}{P-T-S+R} \right) = \left(\frac{D_r}{D_g - D_r} \quad \frac{-D_g}{D_r - D_g} \right)$

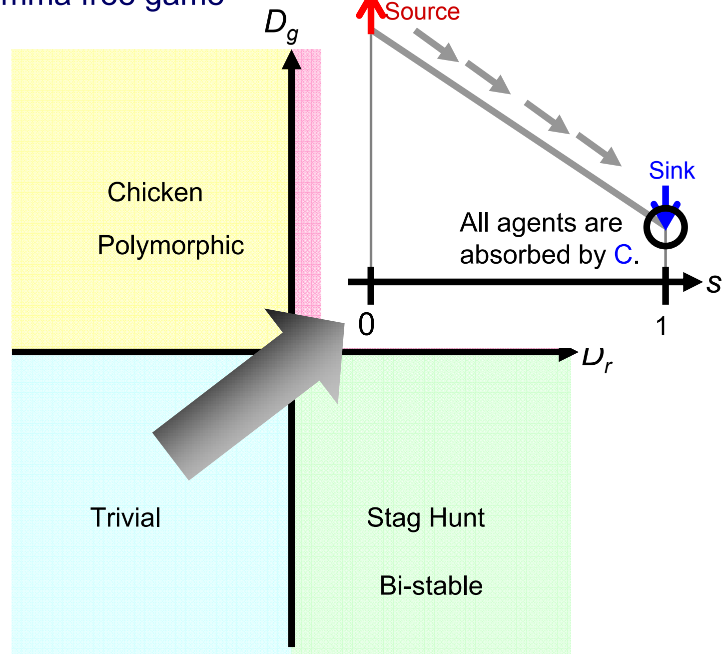

Phase diagram of 2 by 2 games
D_g
D_r
Chicken
PD
Trivial
Stag Hunt


Prisoner's Dilemma, PD
D_g
D_r
Chicken
PD
Stag Hunt
Source
Sink
All agents are
absorbed by D.
s
0
1

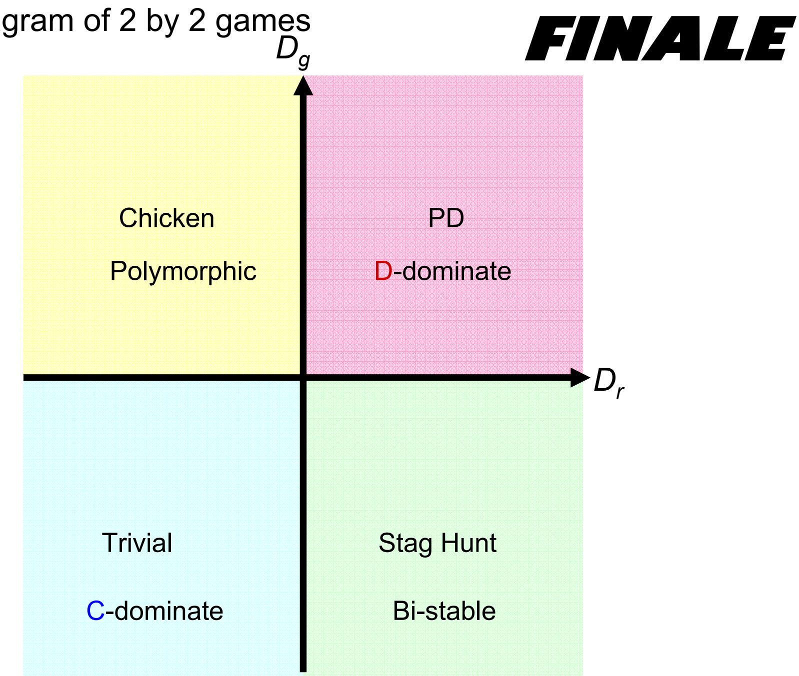

Chicken
D_g
Chicken
PD
D-dominate
Trivial
Source
Source
Sink
All agents are absorbed by Internal Equilibrium.
0
1
s


Stag Hunt
D_g
Source
Sink
Sink
Depending on initial distribution, some agents are absorbed by D and other are absorbed by C.
0
1
s
PD
D-dominate
D_r
Trivial
Stag Hunt

Trivial, dilemma free game



Phase diagram of 2 by 2 games



Backgrounds & Purpose

Most previous studies

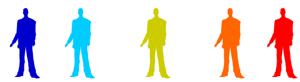
Entirely cooperation ⟷ Entirely defection

Agents can offer either cooperation (C) or defection (D)

Discrete strategy

The real world

Entirely cooperation ⟷ Entirely defection



Actual options might be continuous rather than discrete

Continuous or mixed strategy

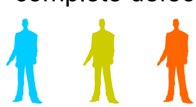
One crucial question is whether there is a considerable difference in game equilibria between the continuous or mixed strategies and those of discrete strategies?

Setting for continuous, and mixed strategy games

Continuous strategy

1. Strategy value: $s_i \in [0,1]$

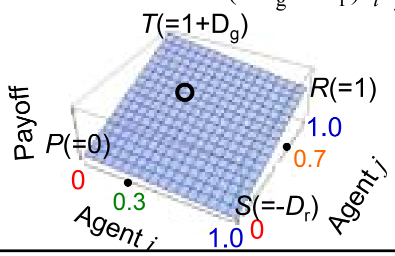
$s_i=1$ complete cooperation
$s_i=0$ complete defection



(0.8) (0.5) (0.2)

2. Payoff function

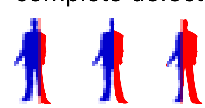
$$\pi(s_i,s_j) \equiv -D_r s_i + (1+D_g)s_j + (-D_g+D_r)s_i s_j$$



Mixed strategy

1. Strategy value: $s_i \in [0,1]$

$s_i=1$ complete cooperation
$s_i=0$ complete defection



(0.8) (0.5) (0.2)

Agents can only offer either C or D according to this strategy

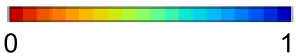
C when Rnd[] < s_i, otherwise *D*
Rnd[ ]: a random number

2. Payoff function

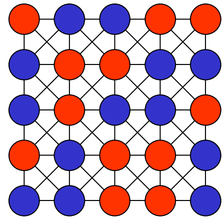
Agent *i* \ Agent *j*	C	D
C	1, 1	$-D_r$, $1+D_g$
D	$1+D_g$, $-D_r$	0, 0

Results

Averaged cooperation fraction



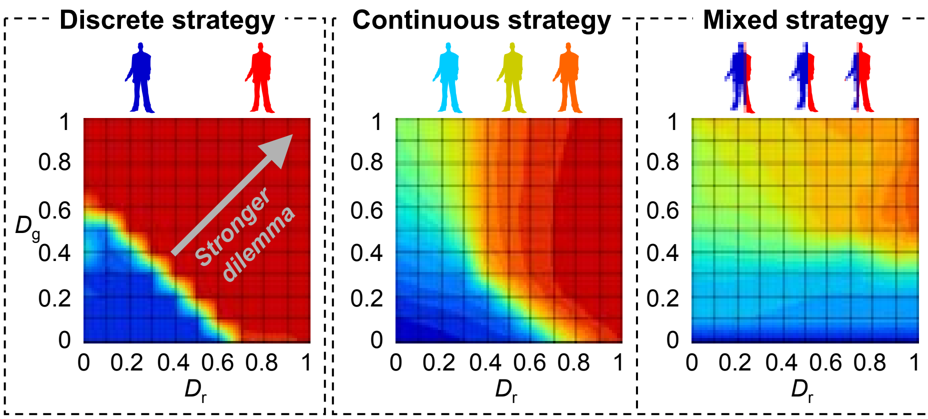
Games are played on lattices (k = 8)



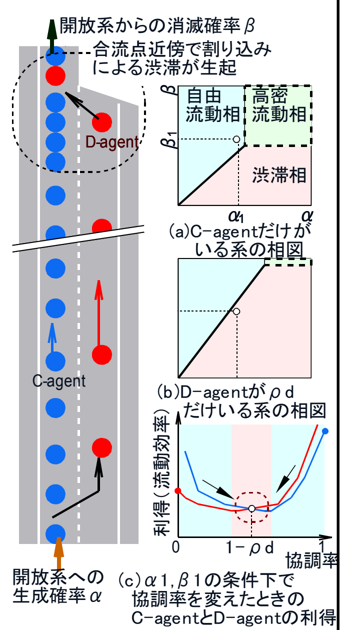
	C	D
C	1	$-D_r$
D	$1+D_g$	0

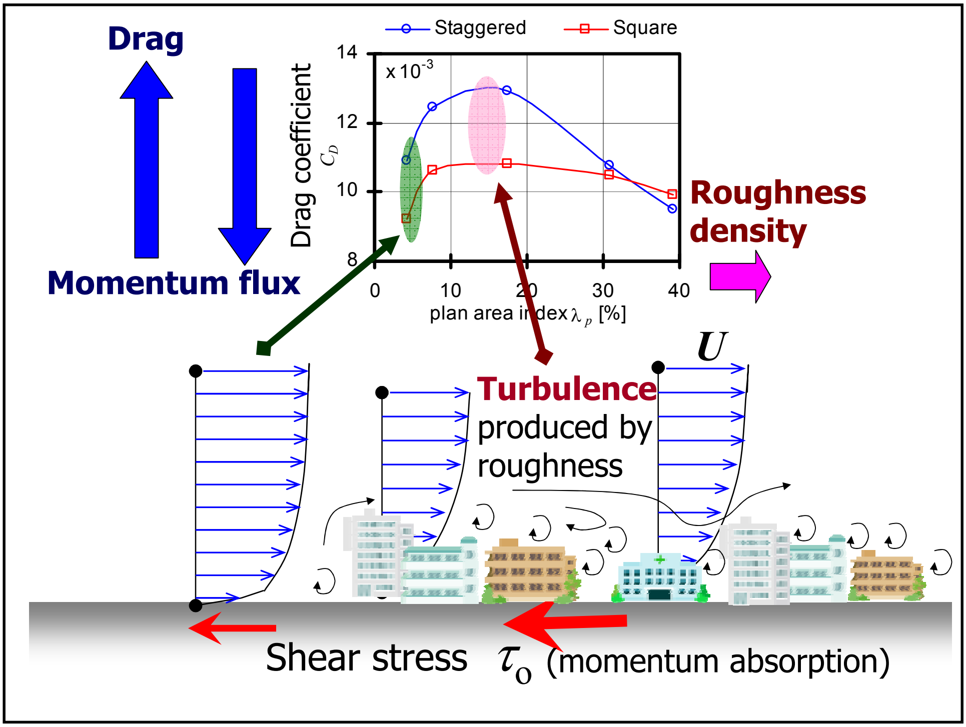
D_g; GID D_r; RAD

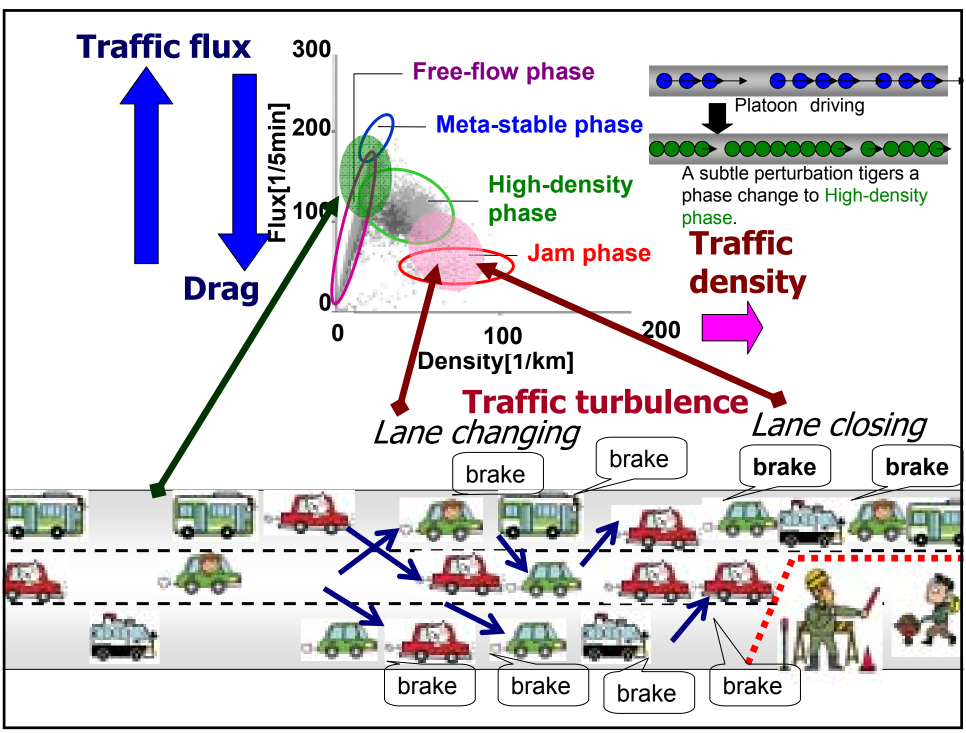
C D

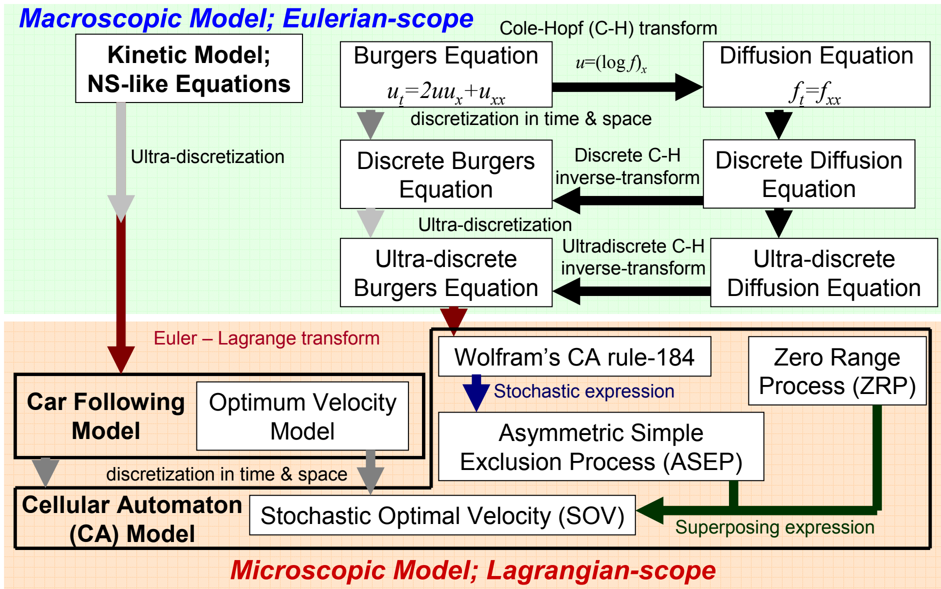


Dilemma game structure hidden in traffic flow at a bottleneck due to a 2 into 1 lane junction

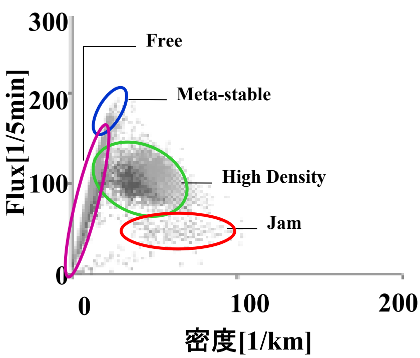



Drag
Momentum flux
Staggered
Square
x 10^-3
Drag coefficient C_D
14
12
10
8
0
10
20
30
40
plan area index λ_p [%]
Roughness density
U
Turbulence
produced by roughness
Shear stress τ_o (momentum absorption)

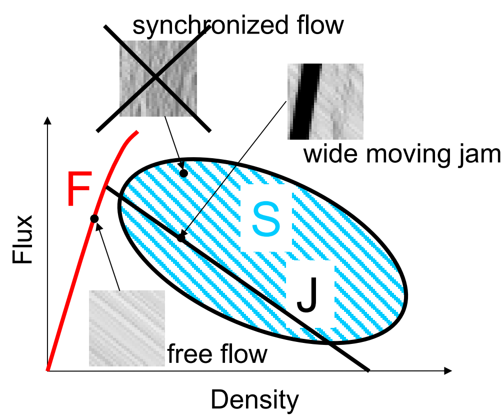

Traffic flux
Drag
300
200
100
0
Flux[1/5min]
0
100
200
Density[1/km]
Free-flow phase
Meta-stable phase
High-density phase
Jam phase
Platoon driving
A subtle perturbation tigers a phase change to High-density phase.
Traffic density
Traffic turbulence
Lane changing
Lane closing
brake
brake
brake
brake
brake
brake
brake
brake



Real Traffic flow

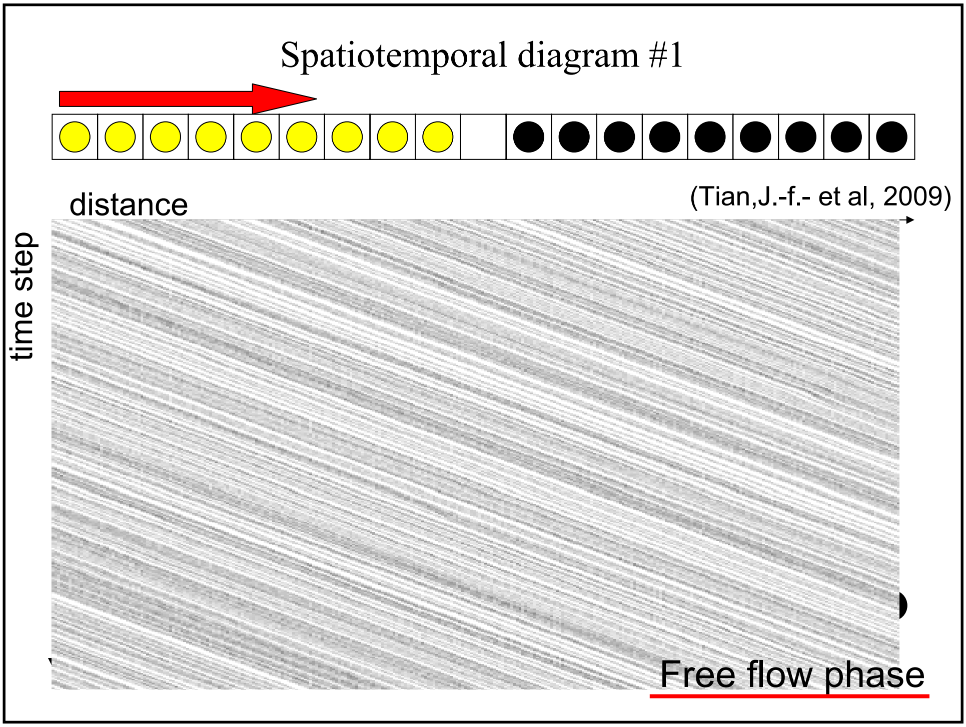


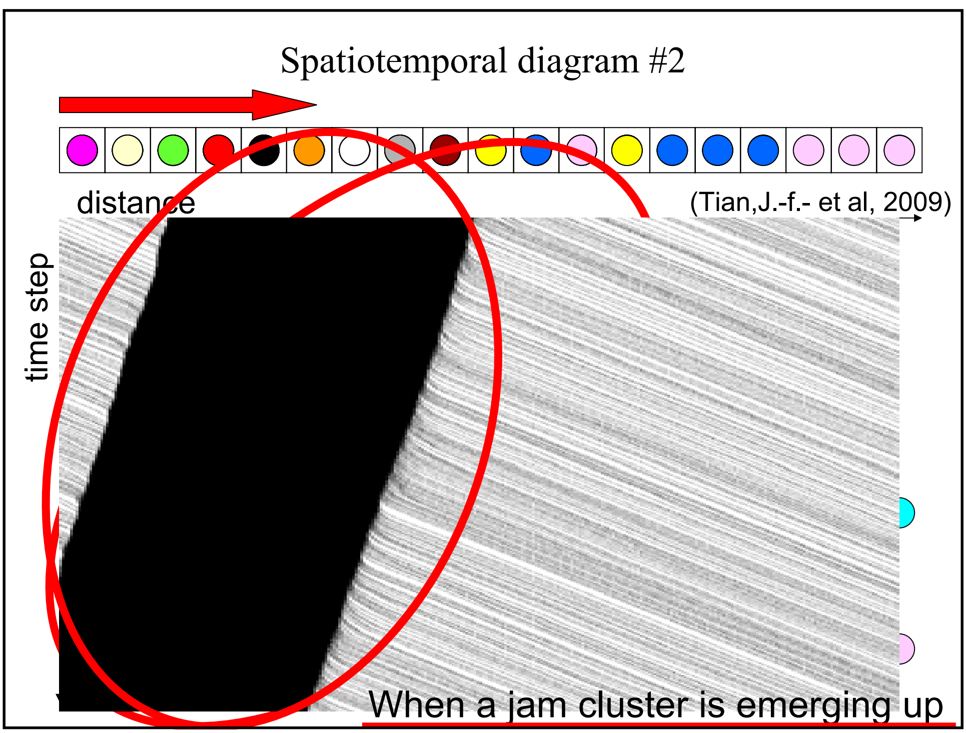
Observed at Tomei highway (Sugiyama et al.)

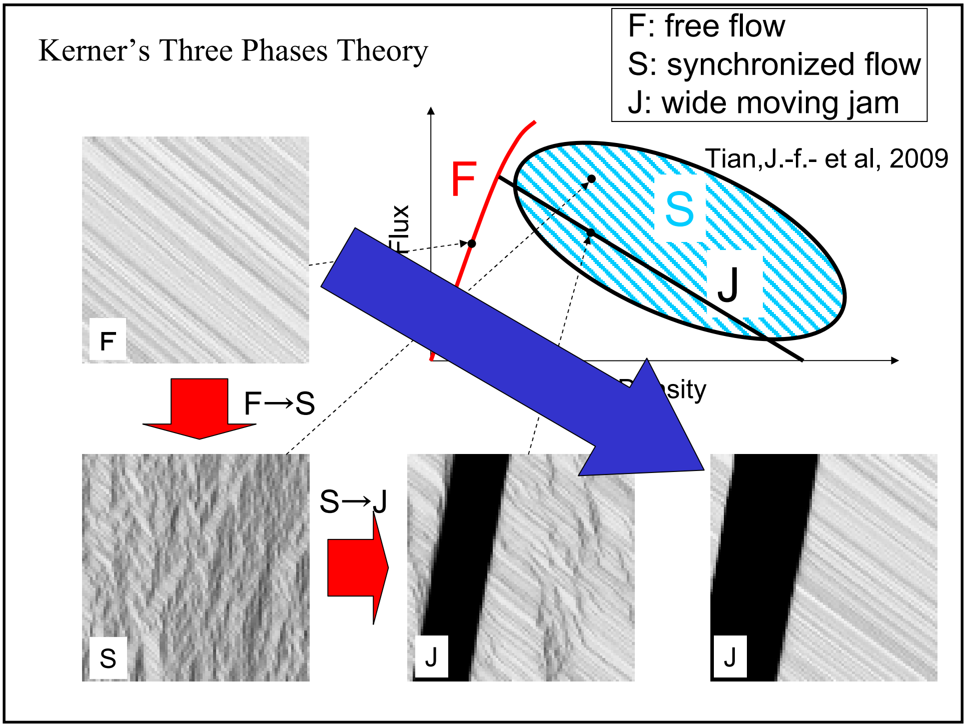


F: free flow
S: synchronized flow
J: wide moving jam

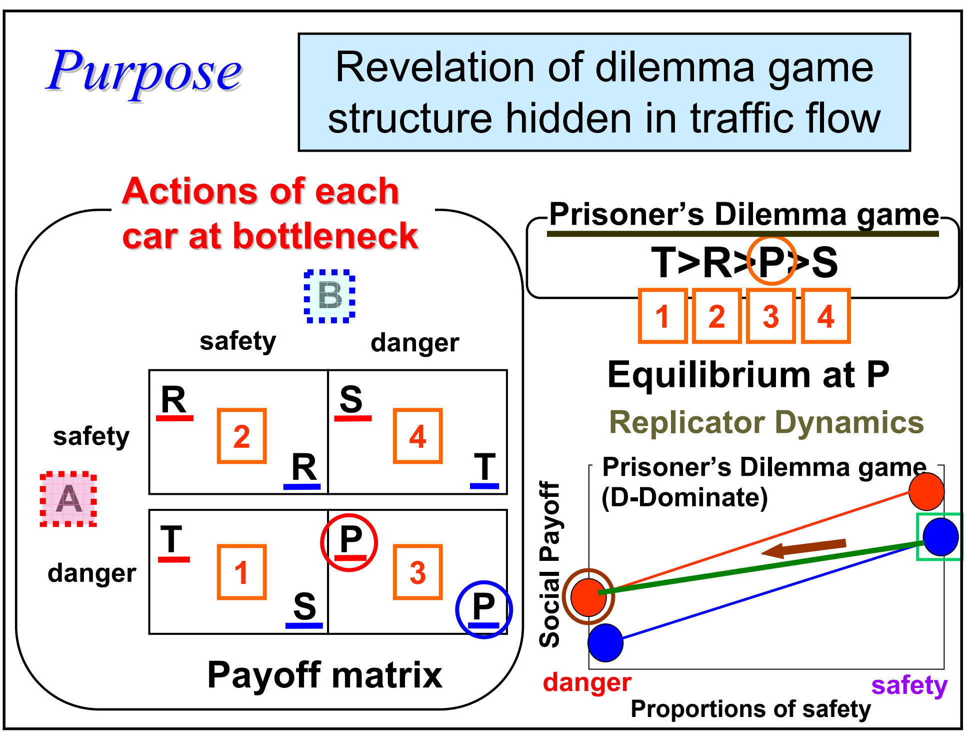
Kerner's Three Phases Theory


Spatiotemporal diagram #1
distance
(Tian,J.-f.- et al, 2009)
time step
Free flow phase

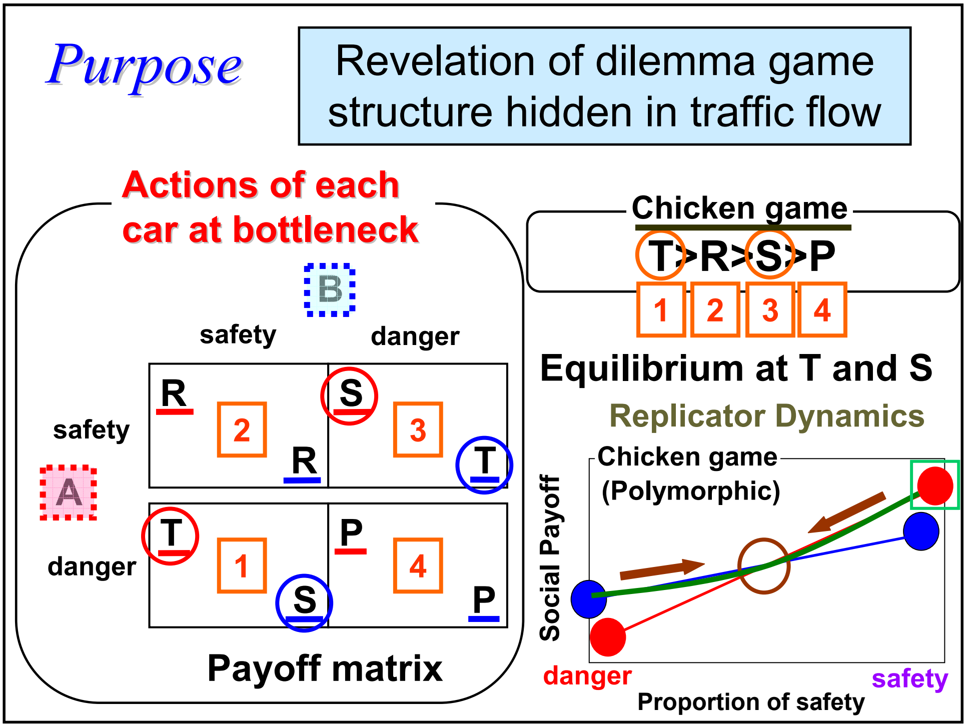

Spatiotemporal diagram #2
distance
(Tian,J.-f.- et al, 2009)
time step
When a jam cluster is emerging up



Kerner's Three Phases Theory
F: free flow
S: synchronized flow
J: wide moving jam
Tian,J.-f.- et al, 2009
F
S
J
Flux
F
F→S
S→J
S
J
J



Purpose
Revelation of dilemma game structure hidden in traffic flow
Actions of each car at bottleneck
B
safety
danger
safety
danger
A
R
2
R
S
4
T
T
1
S
P
3
P
Payoff matrix
Prisoner's Dilemma game
T>R>P>S
1
2
3
4
Equilibrium at P
Replicator Dynamics
Prisoner's Dilemma game
(D-Dominate)
Social Payoff
danger
safety
Proportions of safety



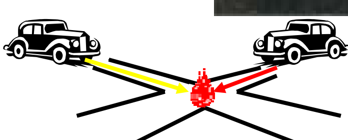
Chicken Game ／ **Hawk–Dove Game** (Maynard Smith (1982)) ／ **Snowdrift Game**

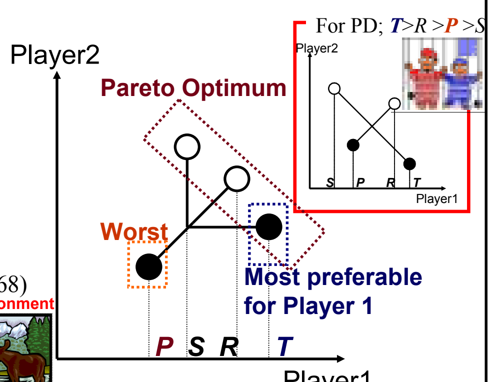
2 x 2 game

Player2 / Player1	C	D
C	4, 4	3, 5
D	5, 3	2, 2

$$\begin{bmatrix} R & S \\ T & P \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}$$

For Chicken; $T > R > S > P$





Max is ***T*** (exploiting) and Min is ***P*** (mutually defecting).

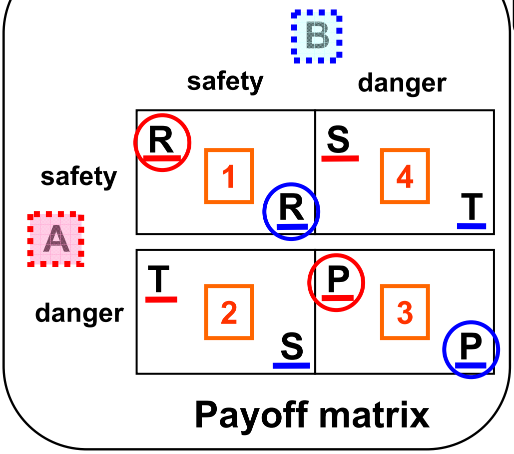
This seems Chicken is a good metaphor for "resource-competing problems"

N-Chicken = Tragedy of Commons (Hardin, 1968)

Purpose

Revelation of dilemma game structure hidden in traffic flow

Actions of each car at bottleneck



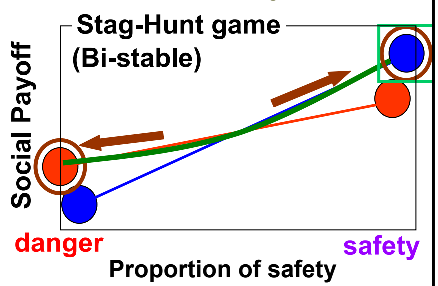
Stag-Hunt game

R>T>P>S

1 2 3 4

Equilibrium at R and P

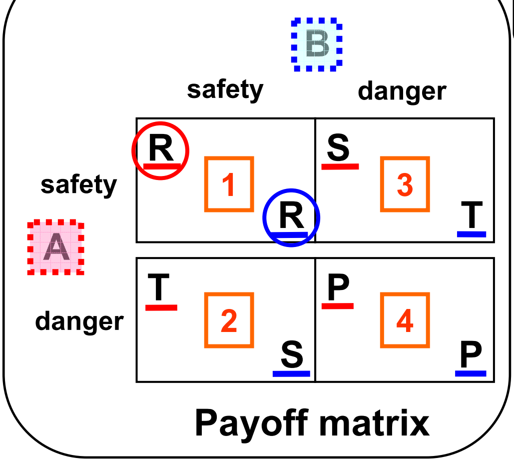
Replicator Dynamics



Purpose

Revelation of dilemma game structure hidden in traffic flow

Actions of each car at bottleneck



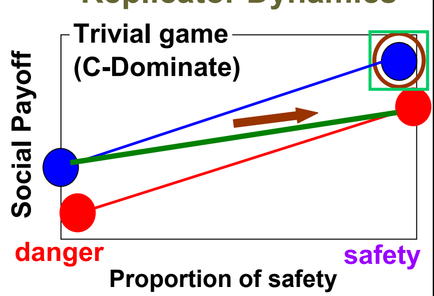
Trivial game

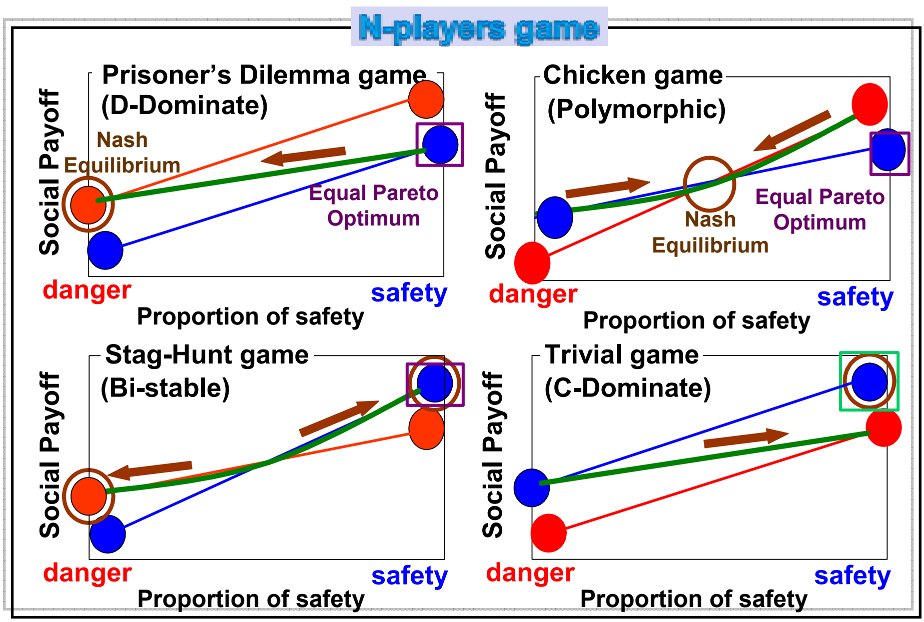
R>T>S>P

1 2 3 4

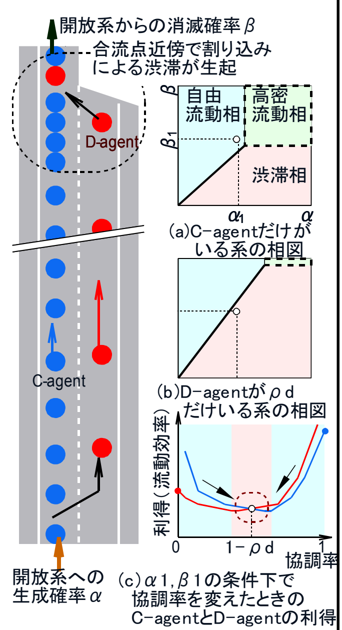
Equilibrium at R

Replicator Dynamics





Dilemma game structure hidden in traffic flow at a bottleneck due to a 2 into 1 lane junction



Backgournd

Trafic flow

Simulation analysis by modeling

•Macrosco

Kinetic g

•Microscop

Dynamical approach by self-driven multiparticle system

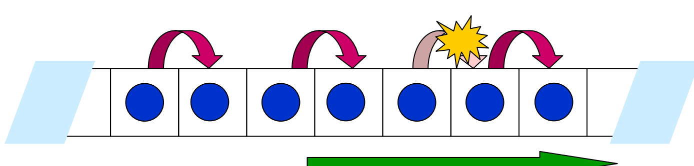
Decision-making process has not been considered.

Game theory

Decision-making process on situation indivisuals exist.

The cellular automaton (CA) model

- ❑Discretization of time and space
- ❑Discretization of property
- ❑Rules for dynamics
- ❑Effect of excluding volume



Stochastic Optimal Velocity (SOV) model

Velocity in the SOV model

$$v_i^{t+1} = (1-a)v_i^t + aV_i^t(\Delta x)$$

Inertia — $(1-a)v_i^t$

Acceleration and Deceleration — $aV_i^t(\Delta x)$

v: velocity

a: parameter

Δx: headway

V: optimal velocity function

Yamauchi *et al.*; Phys. Rev. E 79, #036104 (2009).

SOV model can't reproduce dynamics of traffic flow in detail.

S-NFS model

S-NFS model can reproduce realistically plausible traffic flows.

S-NFS model

If probability r true $s_i = S$, else $s_i = 1$

$$v_i^{(1)} = \min\{V_{\max}, v_i^{(0)} + 1\}$$ — Acceleration

$$v_i^{(2)} = \min\{v_i^{(1)}, x_{i+s_i}^{t-1} - x_i^{t-1} - s_i\}$$ — The slow-to-start effect

calling by probability q

$$v_i^{(3)} = \min\{v_i^{(2)}, x_{i+s_i}^{t} - x_i^{t} - s_i\}$$ — Perspective effect

$$v_i^{(4)} = \max\{0, v_i^{(3)} - 1\}$$ — Random braking

calling by probability 1-p

$$v_i^{(5)} = \min\{v_i^{(4)}, x_{i+1}^{t} - x_i^{t} - 1 + v_{i+1}^{(4)}\}$$ — Collision avoidance

$$x_i^{t+1} = x_i^t + v_i^{(5)}$$ — Renewing car's location

S-NFS model

If probability r true $s_i=S$, else $s_i=1$

$$v_i^{(1)} = \min\{V_{\max}, v_i^{(0)} + 1\}$$ Acceleration

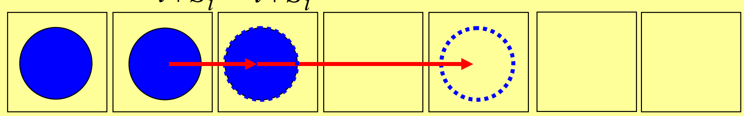
$$v_i^{(2)} = \min\{v_i^{(1)}, x_{i+s_i}^{t-1} - x_i^{t-1} - s_i\}$$ The slow-to-start effect

$v_i^{(3)}$ … Perspective …

v_i …

$x_i^{t+1} = x_i^t$ … location

When $t'=t+1$



S-NFS model

If probability r true $s_i=S$, else $s_i=1$

$$v_i^{(1)} = \min\{V_{\max}, v_i^{(0)} + 1\}$$ Acceleration

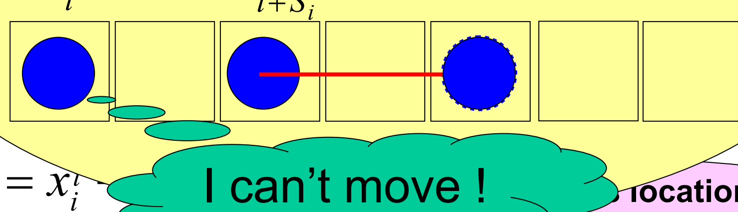
$$v_i^{(2)} = \min\{v_i^{(1)}, x_{i+s_i}^{t-1} - x_i^{t-1} - s_i\}$$ The slow-to-start effect

$v_i^{(3)}$ … Perspective …

v_i …

$x_i^{t+1} = x_i^t$ … location

When $t'=t+1$



S-NFS model

If probability r true s_i=S, else s_i=1

$$v_i^{(1)} = \min\{V_{\max}, v_i^{(0)} + 1\}$$ — Acceleration

$$v_i^{(2)} = \min\{v_i^{(1)}, x_{i+s_i}^{t-1} - x_i^{t-1} - s_i\}$$ — The slow-to-start effect

calling by probability q

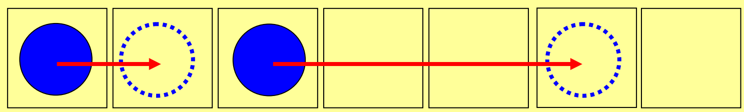
$v_i^{(3)} = \min\{v^{(2)}, x^t \ldots$ — Perspective effect

$v_i^{(4)} \ldots$

$x_i^{t+1} \ldots$ — …tion

No perspective effect（s_i=1）

x_i^t $x_{i+s_i}^t$



S-NFS model

If probability r true s_i=S, else s_i=1

$$v_i^{(1)} = \min\{V_{\max}, v_i^{(0)} + 1\}$$ — Acceleration

$$v_i^{(2)} = \min\{v_i^{(1)}, x_{i+s_i}^{t-1} - x_i^{t-1} - s_i\}$$ — The slow-to-start effect

calling by probability q

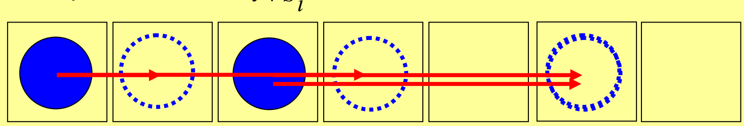
$v_i^{(3)} = \min\{v^{(2)}, x^t \ldots$ — Perspective effect

$v_i^{(4)} \ldots$

$x_i^{t+1} \ldots$ — …tion

No perspective effect（s_i=1）
Perspective effect（s_i=2）

x_i^t $x_{i+s_i}^t$



S-NFS model

If probability r true $s_i = S$, else $s_i = 1$

$$v_i^{(1)} = \min\{V_{\max}, v_i^{(0)} + 1\}$$ — **Acceleration**

$$v_i^{(2)} = \min\{v_i^{(1)}, x_{i+s_i}^{t-1} - x_i^{t-1} - s_i\}$$ — **The slow-to-start effect**

calling by probability q

$$v_i^{(3)} = \min\{v_i^{(2)}, x_{i+s_i}^{t} - x_i^{t} - s_i\}$$ — **Perspective effect**

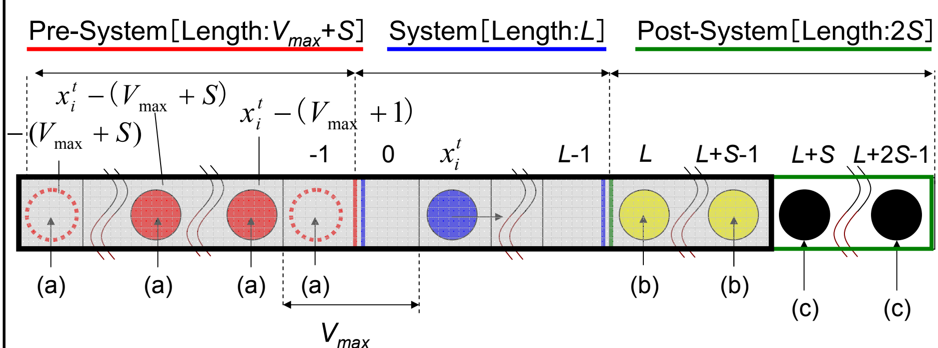
$$v_i^{(4)} = \max\{0, v_i^{(3)} - 1\}$$ — **Random braking**

calling by probability 1-p

$$v_i^{(5)} = \min\{v_i^{(4)}, x_{i+1}^{t} - x_i^{t} - 1 + v_{i+1}^{(4)}\}$$ — **Collision avoidance**

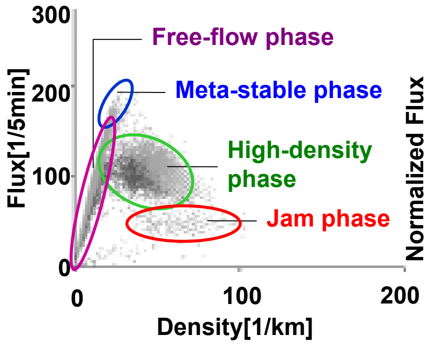
$$x_i^{t+1} = x_i^{t} + v_i^{(5)}$$ — **Renewing car's location**

Open boundary condition



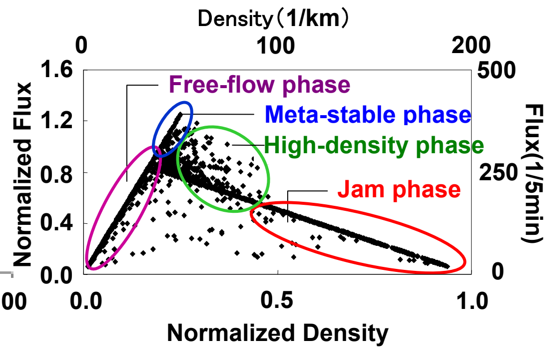
- (blue) The last car in the System (location x_i^t)
- (red) Each car is injected with probability α. (a)
- (yellow) Each car is injected with probability 1-β. (b)
- (black) Each car is always existing in the Post-system. (c)

Reproduction fundamental diagram at a single-lane



Fundamental diagram of real traffic flow

Y.Sugiyama; Nagare 22, 95 (2003)

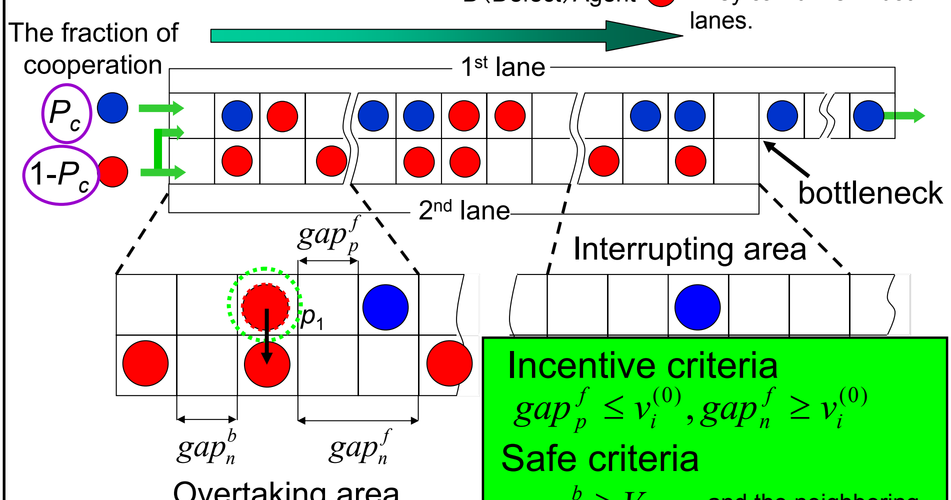


Fundamental diagram of S-NFS model with open boundary condition (p=0.99,q=0.99,r=0.99)

2 lane model

C（Cooperate）Agent — They drive only in the 1st lane.

D（Defect）Agent — They can drive in both lanes.



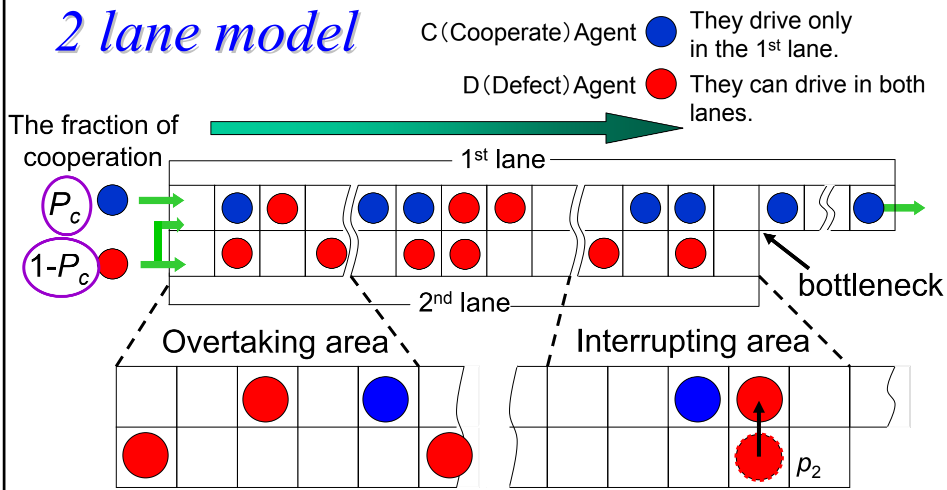
Incentive criteria

$$gap_p^f \leq v_i^{(0)}, gap_n^f \geq v_i^{(0)}$$

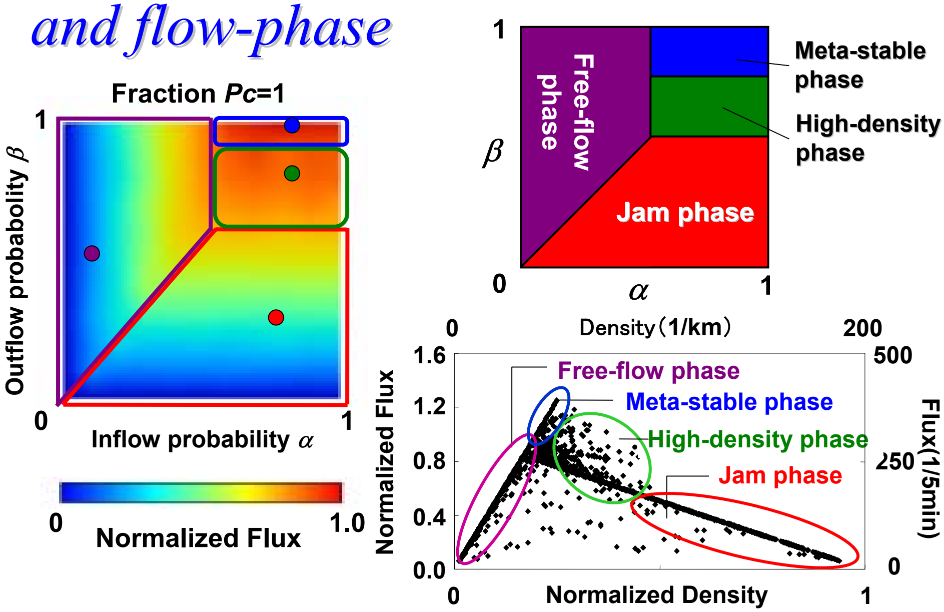
Safe criteria

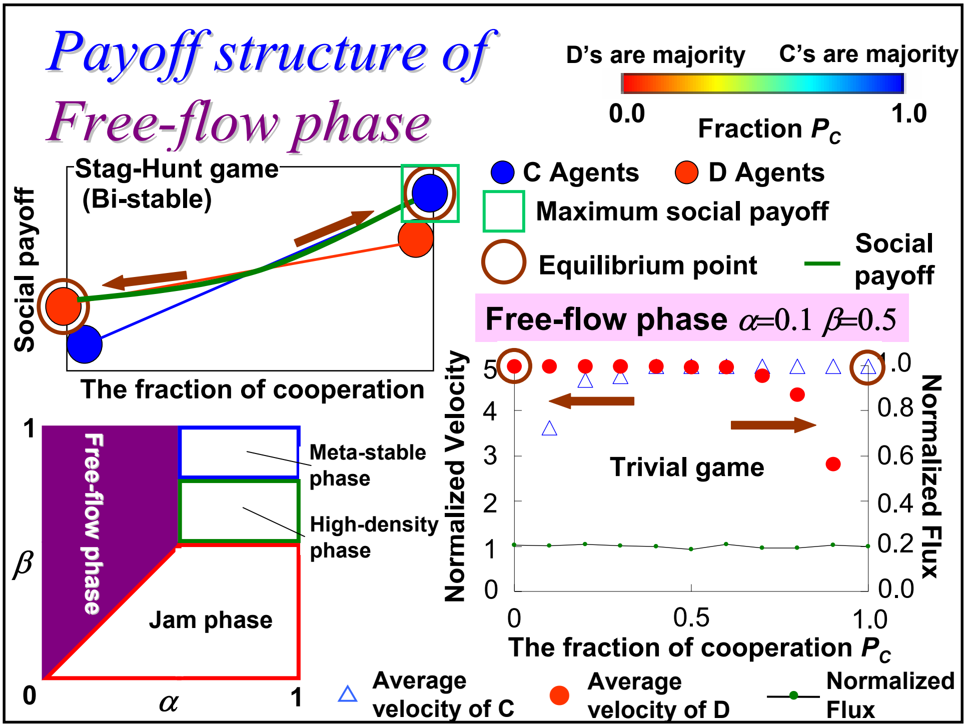
$$gap_n^b \geq V_{\max}$$ and the neighboring cell in the first lane is unoccupied.

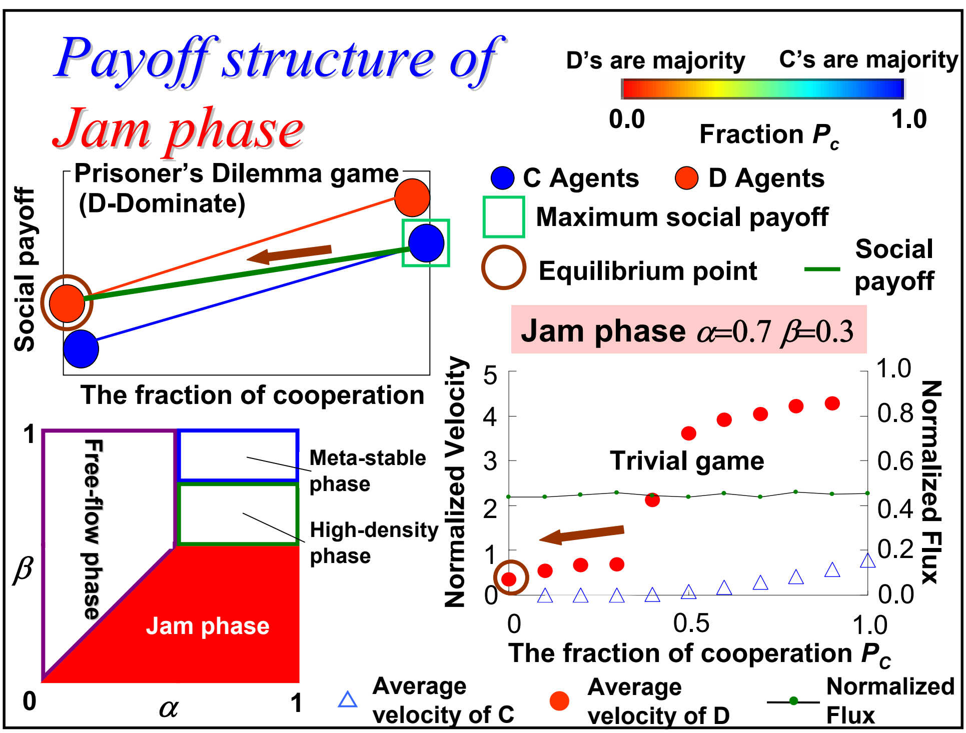
2 lane model

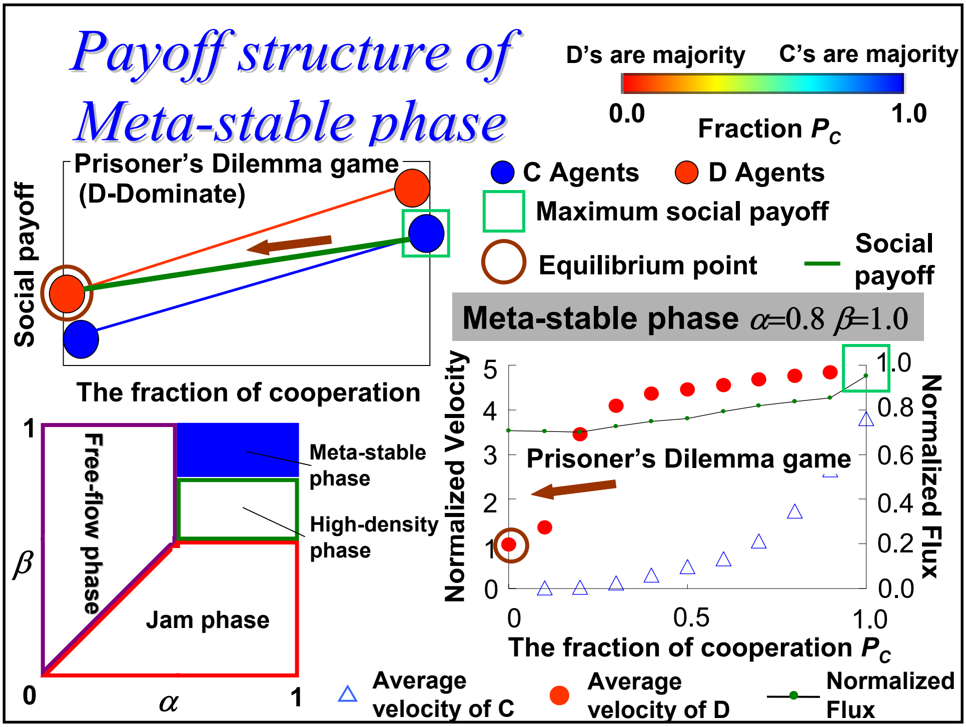


Relation between α–β and flow-phase

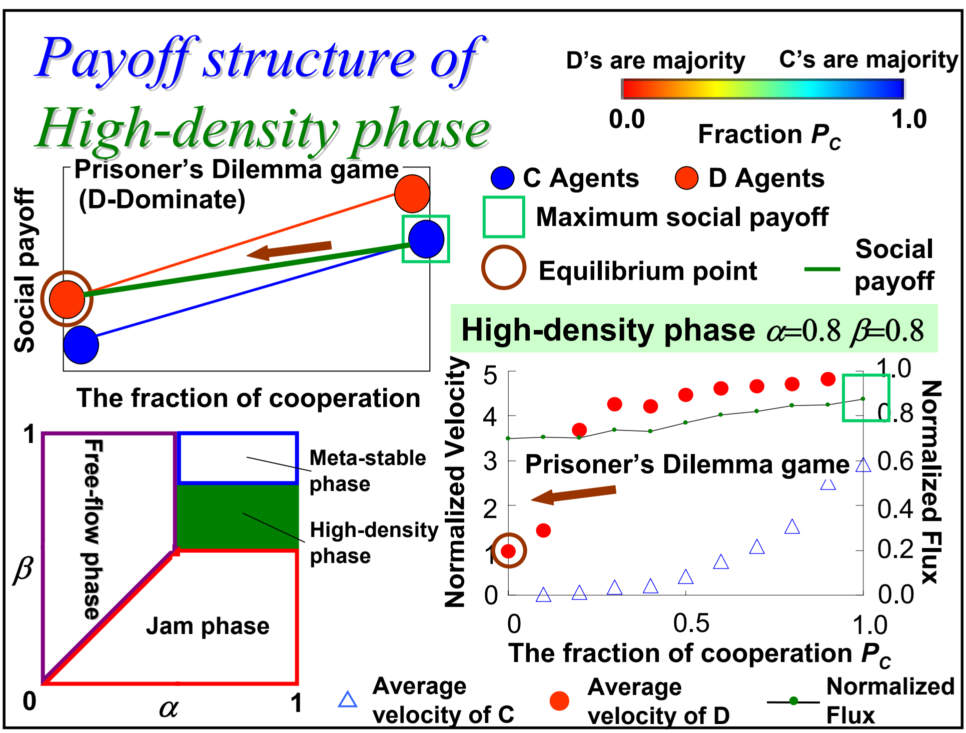



Payoff structure of
Free-flow phase
D's are majority
C's are majority
0.0
Fraction P_c
1.0
Stag-Hunt game
(Bi-stable)
Social payoff
The fraction of cooperation
C Agents
D Agents
Maximum social payoff
Equilibrium point
Social payoff
Free-flow phase α=0.1 β=0.5
Normalized Velocity
Normalized Flux
Trivial game
The fraction of cooperation P_c
Free-flow phase
Meta-stable phase
High-density phase
Jam phase
β
α
Average velocity of C
Average velocity of D
Normalized Flux


Payoff structure of
Jam phase
D's are majority
C's are majority
0.0
Fraction P_c
1.0
Prisoner's Dilemma game
(D-Dominate)
Social payoff
The fraction of cooperation
C Agents
D Agents
Maximum social payoff
Equilibrium point
Social payoff
Jam phase α=0.7 β=0.3
Normalized Velocity
Normalized Flux
Trivial game
The fraction of cooperation P_c
Free-flow phase
Meta-stable phase
High-density phase
Jam phase
β
α
Average velocity of C
Average velocity of D
Normalized Flux



Payoff structure of
Meta-stable phase
D's are majority
C's are majority
0.0
Fraction P_C
1.0
Prisoner's Dilemma game
(D-Dominate)
Social payoff
The fraction of cooperation
C Agents
D Agents
Maximum social payoff
Equilibrium point
Social payoff
Meta-stable phase α=0.8 β=1.0
Free-flow phase
Meta-stable phase
High-density phase
Jam phase
β
α
Normalized Velocity
Normalized Flux
Prisoner's Dilemma game
The fraction of cooperation P_C
Average velocity of C
Average velocity of D
Normalized Flux

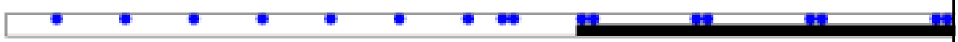


Payoff structure of
High-density phase
D's are majority
C's are majority
0.0
Fraction P_C
1.0
Prisoner's Dilemma game
(D-Dominate)
Social payoff
The fraction of cooperation
C Agents
D Agents
Maximum social payoff
Equilibrium point
Social payoff
High-density phase α=0.8 β=0.8
Free-flow phase
Meta-stable phase
High-density phase
Jam phase
β
α
Normalized Velocity
Normalized Flux
Prisoner's Dilemma game
The fraction of cooperation P_C
Average velocity of C
Average velocity of D
Normalized Flux

Motion of cars at a bottleneck

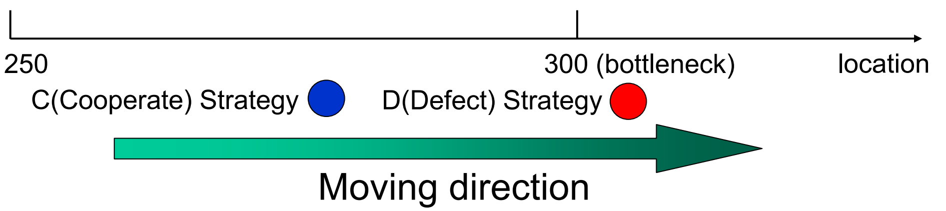
Meta-stable phase (α=1.0, β=1.0) Indication from time step t=3000

・C's are majority and D's are minority. (Fraction P_C=0.9) Normalized Flux: 0.877



・C's and D's are half. (Fraction P_C=0.5) Normalized Flux: 0.744

・D's only exist. (Fraction P_C=0.0) Normalized Flux: 0.701

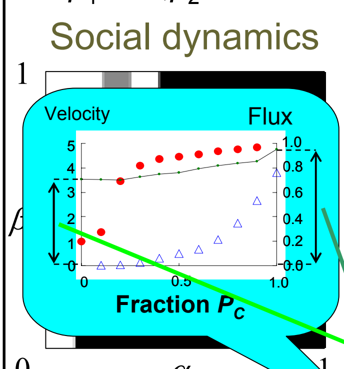


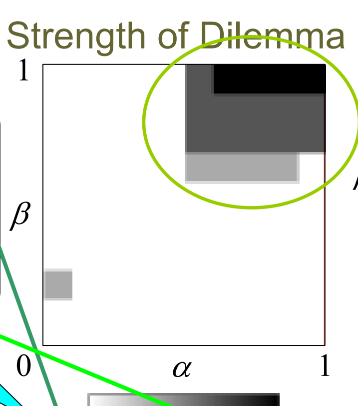
Strength of dilemma on each phase

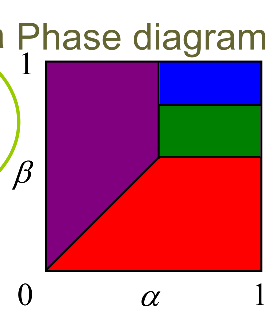
D's are majority C's are majority

0.0 Fraction P_c 1.0

p_1=0.6,p_2=0.8







Free-flow phase

Jam phase

High-density phase

Meta-stable phase

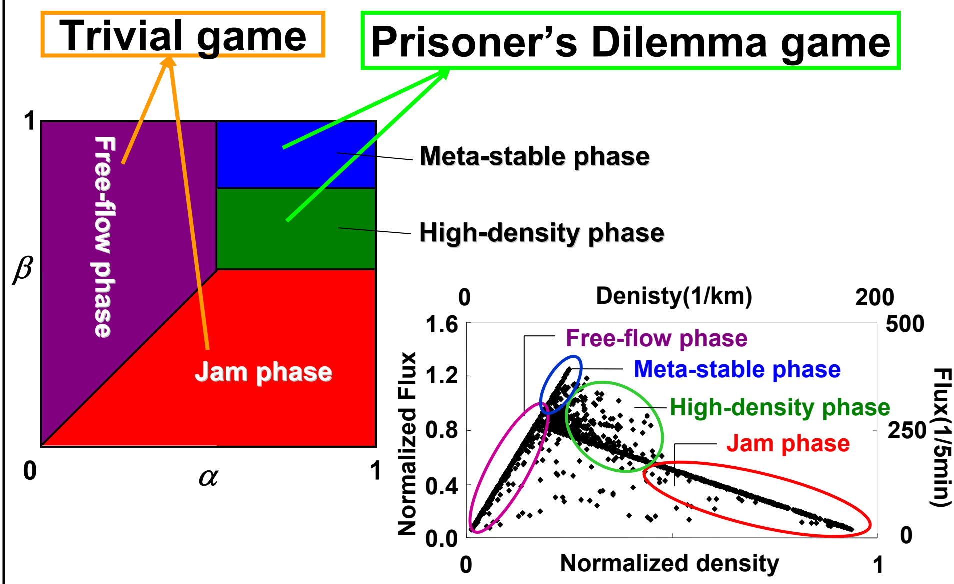
Black: D-dominate at all Pc

Gray: C-dominate at all Pc

White: the rest

$$n = \frac{flux|_{Pc=1.0} - flux|_{Pc=0.0}}{flux|_{Pc=1.0}}$$

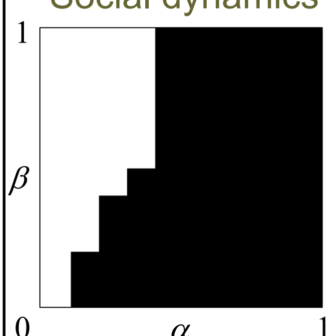
Game structure on each phase



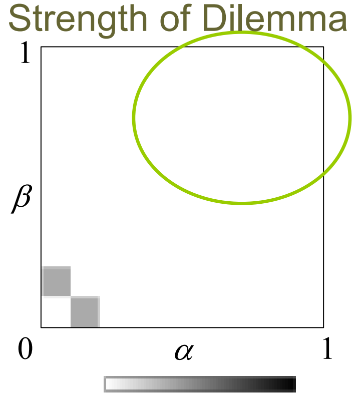
Strength of dilemma when it's hard for cars to cut into the lane

p_1=0.6, p_2=0.1

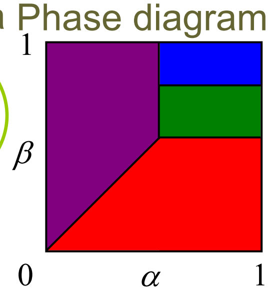
Social dynamics



Strength of Dilemma



Phase diagram

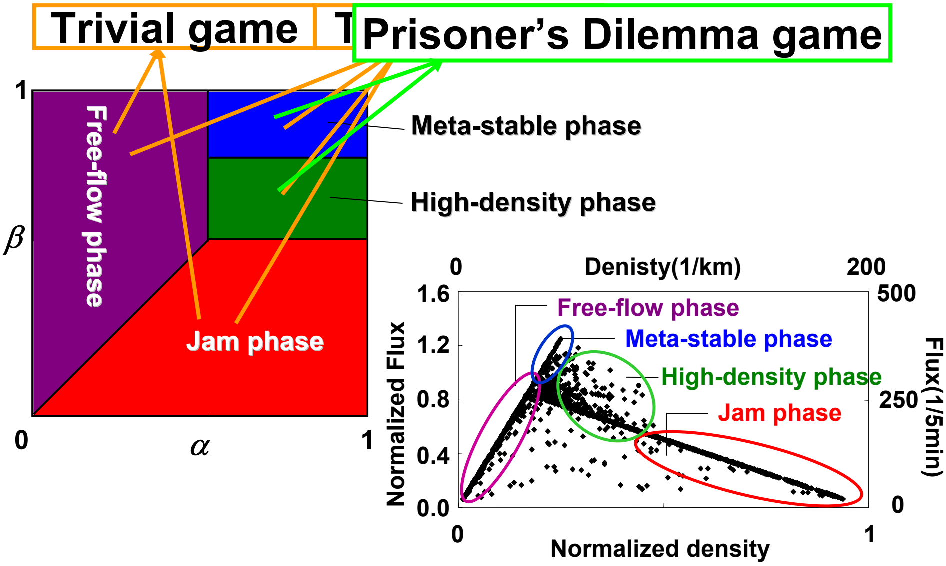


Free-flow phase
Jam phase
High-density phase
Meta-stable phase

Black: D-dominate at all *Pc*
Gray: C-dominate at all *Pc*
White: the rest

$$n = \frac{flux|_{Pc=1.0} - flux|_{Pc=0.0}}{flux|_{Pc=1.0}}$$

Game structure when it's hard for cars to cut into the lane



Conclusion

- Free-flow and Jam phase have *Trivial* game structure.
- Meta-stable and High-density phase have *Prisoner's Dilemma* game structure.
- Social dilemma can be diluted by a rigorous traffic rule, in which last minute interruption is never allowed by well mannered drivers.

Future Work

It might be interesting to examine the question of whether frequent lane changes in a 1D-like homogenous road (without any obvious bottlenecks such as a lane-closing, uphill travel, or a tunnel) may also cause another social dilemma. We assume that changing lanes itself could cause a dilemma in a traffic flow.